FÍSICA 3

23/02/10

Coulomb

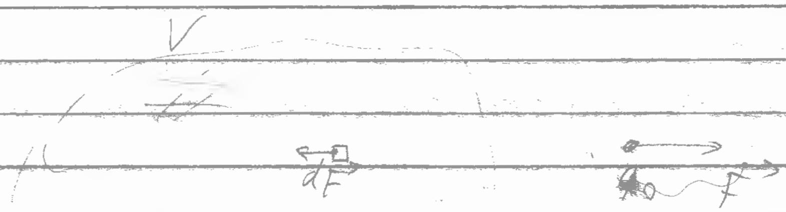
$$\vec{F} = \frac{k q_1 q_2}{r^2}\hat{r} \qquad k = 8{,}98 \cdot 10^{9}\ SI$$

Newton

$$\vec{F} = -\frac{GMm}{r^2}\hat{r} \qquad G = 6{,}67 \cdot 10^{-11}\ SI$$

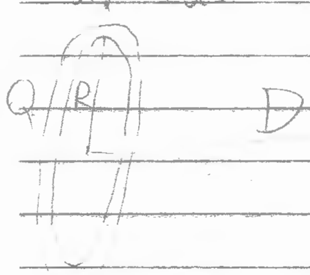
Distribuição Contínua de Carga

$$\rho = \frac{q}{V}$$



$$dF = \frac{k q_0 dq}{r^2}\hat{r} = \frac{k q_0 \rho dV}{r^2}\hat{r}$$

$dl = r d\theta$



$$dF = \frac{k\, dQ\, q}{D^2+R^2} = \frac{k q Q d\theta}{2\pi(D^2+R^2)}$$

$$dF_x = dF \frac{D}{\sqrt{D^2+R^2}} = \frac{k q Q D d\theta}{2\pi(D^2+R^2)^{3/2}}$$

$\lambda = \frac{Q}{2\pi R}$

$$F_x = \frac{kqQ}{2\pi(D^2+R^2)^{3/2}} D \int_0^{2\pi} d\theta$$

$dQ = \lambda dl$

$dQ = \frac{Q}{2\pi} d\theta$

$$F_x = \frac{kqQ}{(D^2+R^2)^{3/2}} D$$

Superposição

DATAPEL

23/02/10

Millikan

1) Carga íon

Fel e Fgrav

Variação discreta

Variação de carga no caminho acelera a part

Concl.: $Q = ne$

Cap 3

Ether

Campo → transmissão de informação

Força ⇒ ação imediata

ação a distância

Campo ⇒ mediador

velocidade finita

Qto tmpo demora p/ A sentir B?

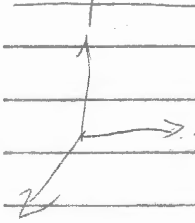
Campo escalar → somente módulo

$(x,y) \to T$ (campo de T)

Campo vetorial → vetor

$(x,y) \to \vec{v}$ (campo de velocidades)

Campo Elétrico



$(x,y,z) \to \vec{E}$

campo vetorial

$\vec{E} = \dfrac{\vec{F}}{q_0}$

q_0 → prova

$[\vec{E}] = N/C$

DATAPEL

24/02/10

"Uma distribuição de cargas no espaço afeta tds os pontos do espaço, produzindo em cada um deles um valor de campo elétrico $\vec{E}$. A carga de prova revela a existência deste campo pela força nela exercida."

Teoricamente: $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0}$

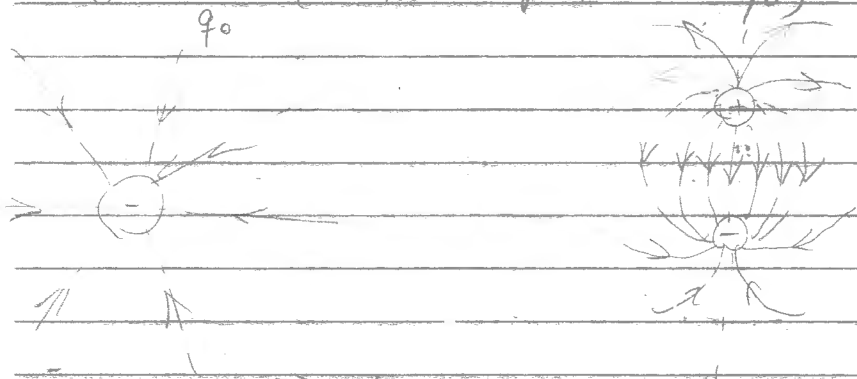
Experimentalmente: $q_0 << Q$

Linhas de Força: visualização

· Direção do campo é // às linhas de força

· $n_{linhas}/A \propto |\vec{E}|$

· $\vec{E} = \frac{\vec{F}}{q_0}$ (sentido depende de q_0)

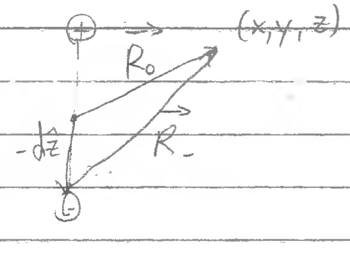


$$\vec{E} = \frac{kq}{r^2}\hat{r}$$

Calculando Campos

Princípio da superposição

· Dipolo elétrico



$$\vec{E}_{\pm} = \frac{\pm kq}{x^2 + y^2 + (z \mp d)^2}\left(x\hat{x} + y\hat{y} + (z \mp d)\hat{z}\right)$$

DATAPEL

24/02/10

$$\vec{E}(x,y,z) = \vec{E}_+ + \vec{E}_-$$

$z = 0$

$$\vec{E}(x,y,0) = \frac{-2kqd}{(x^2+y^2+d^2)^{3/2}}\hat{z} = \frac{-kp}{(x^2+y^2+d^2)^{3/2}}\hat{z}$$

p = momento de dipolo = $2qd$

$x = y = z = 0$

$$\vec{E}(0,0,0) = \frac{-2kq}{d^2}\hat{z}$$

$x = y = 0$

$$\vec{E}(0,0,z) = \frac{kq}{\cancel{z}^2}\left[\left(1-\frac{d}{z}\right)^{-2} - \left(1+\frac{d}{z}\right)^{-2}\right]\hat{z}$$

??

$z >> d$

Taylor

$$\vec{E} = \frac{2kqd}{z^3}\hat{z} = \frac{kp}{z^3}\hat{z}$$

Campo disco no eixo

$$d\vec{E} = \frac{kdQ}{(\rho^2+D^2)^{3/2}}\vec{z}$$

$$dQ = \sigma 2\pi\rho d\rho$$

$$\vec{E} = 2\pi\sigma R\int_0^R \frac{\rho d\rho}{(\rho^2+D^2)^{3/2}}$$

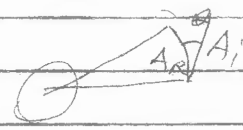
$\vec{E} =$

DATAPEL

$$\oint_S \vec{E}\cdot\hat{n}\, dS = \frac{q_{int}}{\varepsilon}$$

Digressão Sobre Ângulo Sólido

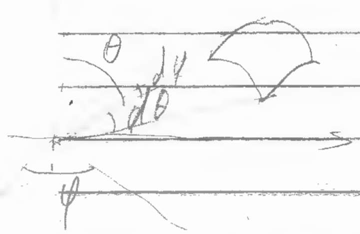
$$\Omega = \frac{A_R}{R^2} \rightarrow \text{esferorad}$$



$$A_R = A_I \cos\theta = A_I\, \hat{R}\cdot\hat{n}$$

$$\Omega = \frac{A\cdot\hat{R}\cdot\hat{n}}{R^2}$$

$$\oint_S \vec{E}\cdot\hat{n}\, dS = \oint_S \frac{kq\hat{R}}{R^2}\cdot\hat{n}\, dS = kq\int \frac{\hat{R}\cdot\hat{n}}{R^2}\, dS = kq\oint d\Omega$$



$$dS = R\,d\theta \,.\, R\,\text{sen}\,\theta\, d\varphi$$

$$dS = R^2\,\text{sen}\,\theta\, d\theta\, d\varphi$$

$$d\Omega = \frac{dS}{R^2} = \text{sen}\,\theta\, d\theta\, d\varphi$$

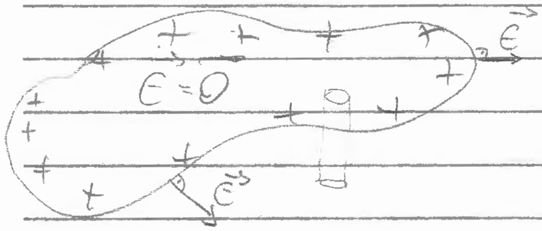
$$\Omega = \int d\Omega = \int_0^{\theta}\int_0^{2\pi} \text{sen}\,\theta\, d\varphi\, d\theta = 1 - \cos\theta$$

Aplicações

- Plano uniformemente carregado

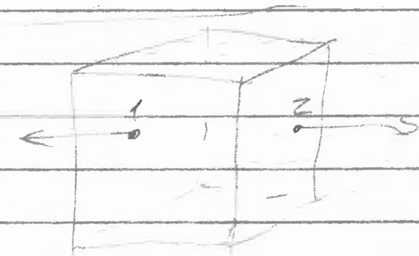
03/03/10

Condutores



$$E \cdot S = \frac{\sigma S}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$



$$\phi_x = A\left[v_x(2) - v_x(1)\right]$$

$$\phi_x = \Delta y \Delta z \left[v_x(2) - v_x(1)\right]$$

$$\phi_y = \frac{\partial v_y}{\partial y} \Delta V$$

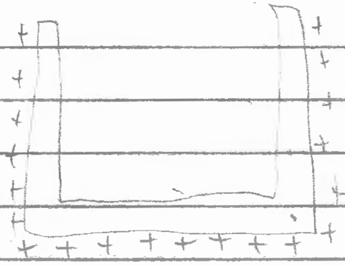
$$\phi = \phi_x + \phi_y + \phi_z = \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\Delta V$$

$$\phi = \oint \vec{v} \cdot d\vec{s}$$

$$\text{div } \vec{v} = \frac{1}{\Delta V} \oint \vec{v} \cdot d\vec{s}$$

$\nexists$ eq estável

carga na superfície



$$\vec{E} \neq \vec{ctc}$$

DATAPEL

03/10

Equação de Poisson

$q_{\Delta\Sigma} = \rho(P) \cdot \Delta V$

$\oint_{\Delta\Sigma} \vec{E} \cdot d\vec{S} = \rho(P) \dfrac{\Delta V}{\varepsilon_0}$

$\dfrac{\rho(P)}{\varepsilon_0} = \dfrac{1}{\Delta V} \int_{\Delta\Sigma} \vec{E} \cdot d\vec{S}$

$\dfrac{\rho(P)}{\varepsilon_0} = \underbrace{\lim_{\Delta V \to 0} \dfrac{1}{\Delta V} \int_{\Delta\Sigma} \vec{E} \cdot d\vec{S}}$

independe $\Delta\Sigma$
depende $\vec{E}$

$\vec{\nabla} \cdot \vec{V} = \operatorname{div} \vec{V} = \lim_{\Delta V \to 0} \dfrac{1}{\Delta V} \oint_{\Delta\Sigma} \vec{E} \cdot d\vec{S}$

$\operatorname{div} \vec{E} = \dfrac{\rho}{\varepsilon_0}$

$\phi_x = \Delta y \Delta z \,(v_x(2) - v_x(1))$

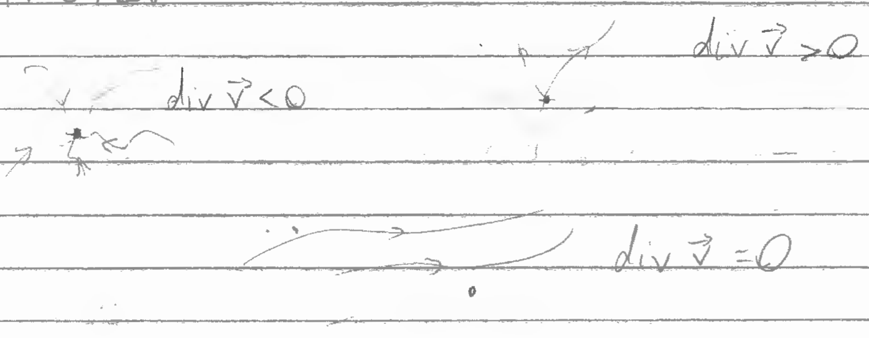
$\phi_x = [v_x(x + \frac{\Delta x}{2}, y, z) - v_x(x - \frac{\Delta x}{2}, y, z)] \cdot \Delta y \Delta z$

$\phi_x = \cancel{v_x(x,y,z)\, \Delta y \Delta z} + \dfrac{\Delta x}{2} \dfrac{\partial}{\partial x} v_x(x,y,z)\, \Delta y \Delta z$

$\quad - \cancel{v_x(x,y,z)\, \Delta y \Delta z} + \dfrac{\Delta x}{2} \dfrac{\partial}{\partial x} v_x(x,y,z)\, \Delta y \Delta z$

$\phi_x = \dfrac{\partial}{\partial x} v_x(x,y,z)\, \Delta V$

$\phi = \operatorname{div} \vec{E} \cdot \Delta V$



11/03/10

$\operatorname{div}\vec{E} = 0$, $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^3}\vec{r}$

$\oint_S \vec{v}.d\vec{s} = \int \operatorname{div}\vec{v}\, dV$

Campos Conservativos

$\omega^{c}_{P_1-P_2} = \int_C \vec{F}\, d\vec{l} = T_2 - T_1$ Depende somente dos ptos iniciais e finais

· Força Central: $\vec{F} = F(r)\,\hat{r}$

$\vec{F}.d\vec{l} = F(r)\,dr$

$\omega = \int_{r_1}^{r_2} F(r)\,dr$ → ñ depende do caminho

$d\vec{l} = d\vec{r} + r\,d\theta$

· Força Conservativa

$\oint_C \vec{F}.d\vec{l} = 0$

$\boxed{\vec{F} = -\operatorname{grad} U = -\vec{\nabla} U}$

Sup equipotencial

$\vec{\nabla} U \perp d\vec{l}$

· $\vec{\nabla} U \parallel d\vec{l} \Rightarrow \max U$

$f = f(r)$

$\operatorname{grad} f = \frac{\partial f}{\partial r} \cdot \operatorname{grad} \vec{r} = \frac{\partial f}{\partial r}\hat{r}$

$\vec{F} \Longrightarrow U$ (Energia potencial)

$\Downarrow \qquad \Downarrow$

$\vec{E} = \frac{\vec{F}}{q_0}$ $\quad V = \frac{U}{q_0}$ (Potencial)

DATAPEL

16/03/10

$U = -\int \vec{F} \cdot \vec{dl}$

$V = \frac{U}{q_0} = -\int \frac{\vec{E}}{q_0} \cdot \vec{dl} = -\int \vec{E} \cdot \vec{dl} \Rightarrow \vec{E} = -\vec{\nabla} V$

Potencial é a (e) Potencial q uma carga unitária teria se fosse trazida de um ponto de referência até um ponto específico do espaço

Energia potencial é a (e) acumulada em um objeto carregado posicionado em um $\vec{E}$

$[U] = J \quad [V] = \frac{J}{C} = V$

eV = (e) necessária para mover um e^- através de uma ddp de 1V

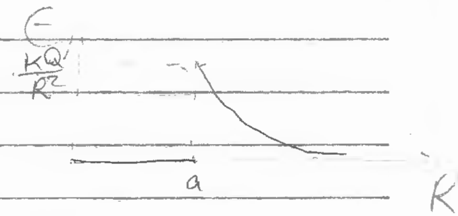
$1eV = 1{,}6 \cdot 10^{-19}\ J$

$\vec{E} \Longleftrightarrow V$

vetor escalar

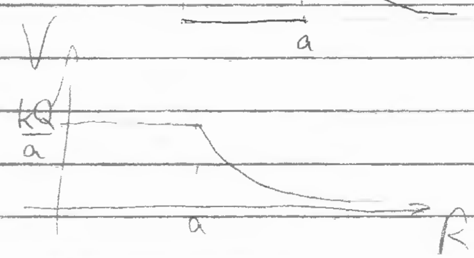
Potencial Coulombiano

$V = \frac{kQ}{R} = -\int \vec{E} \cdot \vec{dl}$



Ex

Casca esférica



Q

a

$\vec{E} = \begin{cases} \frac{kQ}{R^2}\hat{R}, & R > a \\ 0, & R < a \end{cases}$

$V = -\int \vec{E} \cdot \vec{dl}$

$R > a \Rightarrow V = \frac{kQ}{\cancel{R}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$V(\infty) = 0$

$R > a \Rightarrow V = \frac{kQ}{R}$

$R < a \Rightarrow V = \frac{kQ}{a}$

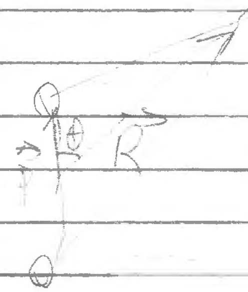
$R < a$

$\vec{E} = -\vec{\nabla} V \Rightarrow V = cte$

17/03/10

Ex

Dipolo Elétrico



$V_T = V_+ + V_- = kq\left(\frac{1}{R_+} - \frac{1}{R_-}\right)$

$R >> l \Rightarrow \begin{cases} R_- - R_+ = l\cos\theta \\ R_- R_+ = R^2 \end{cases}$

$V = \frac{kql\cos\theta}{R^2}$

$\vec{p} = q\vec{l}$

$\vec{p} = ql\hat{z}$

$$\boxed{V = \frac{k|\vec{p}|\cos\theta}{R^2} = \frac{k\vec{p}\cdot\hat{R}}{R^2} = \frac{k\,\vec{p}\cdot\vec{R}}{R^3}}$$

$R >> l$

$V = \frac{k|\vec{p}|z}{R^3}$

$\vec{E}$:

$\vec{E} = -\vec{\nabla} V$

$$\boxed{\vec{E} = -\frac{k\vec{p}}{R^3} + \frac{3kpz}{R^4}\hat{R}}$$

$\frac{z}{R} \to \cos\theta = \hat{z}\cdot\hat{R}$

$$\boxed{\vec{E} = -\frac{k\vec{p}}{R^3} + \frac{3k\vec{p}\cdot\hat{R}}{R^3}\hat{R}}$$

$z = 0 \Rightarrow \vec{p}\cdot\hat{R} = 0 \Rightarrow \vec{E} = -\frac{k\vec{p}}{R^3}$

$x = y = 0 \Rightarrow \vec{p}\cdot\hat{R} = |\vec{p}| \Rightarrow \vec{E} = \frac{2kp}{z^3}\hat{z}$ $(R = z)$

$\hat{R} = \hat{z}$

$$\boxed{\vec{E} = \frac{2kp}{R^3}\hat{z}}$$

DATAPEL

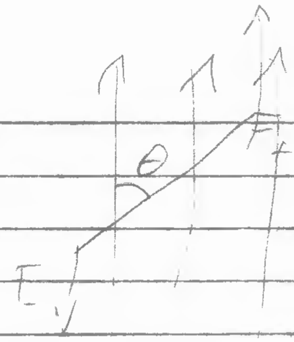
17/03/10

$\vec{\tau} = l F \operatorname{sen}\theta \hat{x}$

$\vec{\tau} = l q E \operatorname{sen}\theta \hat{x}$

$p = ql$

$\vec{\tau} = \vec{p} \times \vec{E}$



Molécula de H_2O é dipolo permanente

Micro-ondas
↳ Luz
↳ radiação eletromag

$U = qV$

$U_T = -\vec{p} \cdot \vec{E}$
↳ ⓔ do dipolo no campo $\vec{E}$

$\vec{F} = -\vec{\nabla} U$

$$\vec{F} = p \frac{\partial E}{\partial x} \hat{x}$$

Gradiente em Coord Polares

$f(x,y)$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$\vec{ds} = dx\hat{x} + dy\hat{y}$$

$$df = \vec{\nabla} f \cdot \vec{ds} = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$\vec{\nabla} f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y}$$

$f(r,\theta)$

$$df = \frac{\partial f}{\partial r} dr + \frac{\partial f}{\partial \theta} d\theta$$

$$\vec{ds} = dr.\hat{r} + d\theta.\hat{\theta}$$

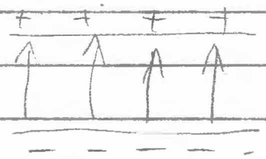
$$\vec{\nabla} f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r} \cdot \frac{\partial f}{\partial \theta}\hat{\theta}$$

DATAPEL

17/03/10

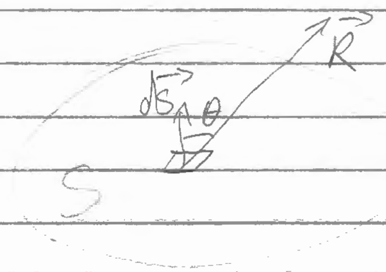
Ex

Superfície de dipolos



σ $\quad dV = \dfrac{k\, d\vec{p} \cdot \hat{R}}{R^2}$ → pot de dipolo

↳ sup de dipolos



$$d\vec{p} = \sigma d\vec{S}$$

$$dV = k \sigma d\Omega$$

$$dV = \frac{k \sigma dS \cos\theta}{R^2}$$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow V > 0$
$\frac{\pi}{2} < \theta < \frac{3\pi}{2} \Rightarrow V < 0$
} descontínuo

Ex

Potencial de Condutores

→ cargas na sup

→ $\vec{E} = 0$ no interior ($V = cte$)

$$\oint_C \vec{E} \cdot d\vec{l} = 0$$

condutor oco

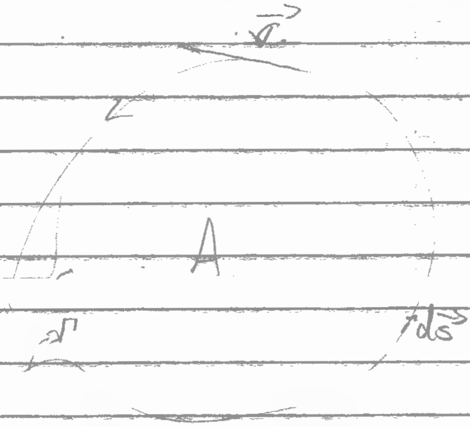
Poliedro

DATAPEL

24/03/10

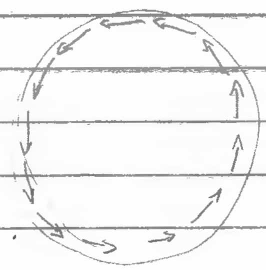
$$\oint \vec{E} \cdot \vec{dl} = 0$$

$$\oint_{\Gamma} \vec{c} \cdot \vec{ds} = \int_{A} (\vec{\nabla} \times \vec{C}) \cdot \hat{n}\, dA$$



$$\vec{\nabla} \times \vec{E} = \vec{0}$$

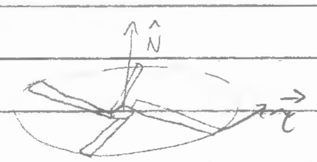
$$\vec{\nabla} \times \vec{V} = 2\vec{\omega}$$



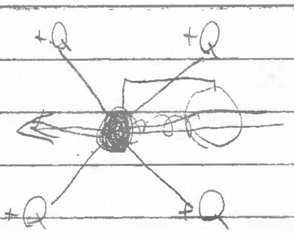
Campo Geral

$$\bar{V}_1 = \frac{1}{2\pi R} \oint_{C} \vec{V} \cdot \vec{\tau}\, dS$$

$$\omega R = \frac{1}{2\pi R} \int_{A} \vec{\nabla} \times \vec{V} \cdot \hat{N}\, dA$$

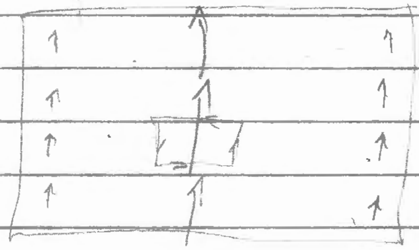


$$2\vec{\omega} = \vec{\nabla} \times \vec{V}\,(x_0, y_0, z_0)$$



24/03/19

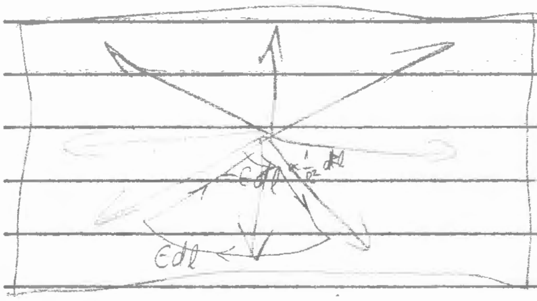
Ex



$\vec{\nabla} \cdot \vec{E} = 0$

$\vec{\nabla} \times \vec{E} \neq 0$

Ex



$\vec{\nabla} \cdot \vec{E} \neq 0 \rightarrow$ fonte

$\vec{\nabla} \times \vec{E} = 0$

$\nabla^2 V = -\dfrac{\rho}{\varepsilon_0}$

$\vec{\nabla}\phi \rightarrow$ taxa variação

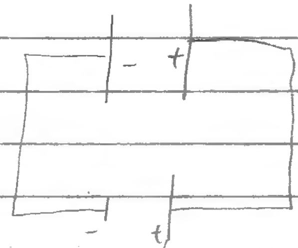
$\vec{\nabla}\vec{V} \rightarrow$ fontes do campo

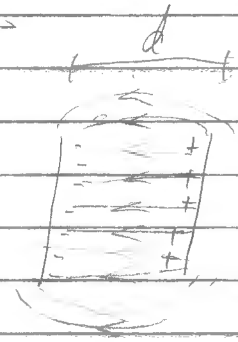
$\vec{\nabla} \times \vec{V} \rightarrow$ taxa rot campo

$\vec{\nabla}(\vec{\nabla} \times \vec{V}) = 0$

Cap 5

Capacitância





$$\int E\,dS = \frac{q}{\varepsilon} = \vec{E}_+ = \frac{\sigma}{2\varepsilon} = \vec{E}_-$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}$$

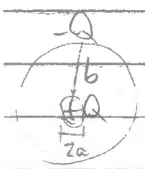
$$\Delta V = \int_+^- \vec{E}\cdot\vec{dl} = Ed = \frac{\sigma d}{\varepsilon_0}$$

$$C = \frac{Q}{\Delta V}$$

Placas // $C = \frac{A\varepsilon_0}{d}$

$[C] = C/V = F$

Cilíndrico



$$R > b \Rightarrow \vec{E} = \vec{0}$$

$$a < R < b \Rightarrow \vec{E} = \frac{Q}{2\pi r l \varepsilon_0}\hat{R}$$

$R <$

$$\Delta V = \int E\,dl = \int_a^b \frac{Q}{2\pi r l \varepsilon_0}\,dR = \frac{Q}{2\pi\varepsilon_0 l}\ln\frac{b}{a}$$

$$C = \frac{2\pi\varepsilon_0 l}{\ln\frac{b}{a}}$$

$$b = a + d \Rightarrow C = \frac{2a\pi\varepsilon_0 l}{d}, \quad d \ll a$$

$$C = \frac{\varepsilon_0 A_{lat}}{d}$$

06/04/10

Esférico

$$\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$$C = \frac{4\pi\varepsilon_0 ab}{b-a}$$

$b - a = d$

$d \ll a \Rightarrow ab = R^2 \Rightarrow C = \frac{\varepsilon_0 A_{esf}}{d}$

$b \gg a \Rightarrow C = 4\pi\varepsilon_0 a$

Associação

// $\rightarrow C = \sum C_i$

série $\rightarrow C^{-1} = \sum C_i^{-1}$

Carregamento

$$V = \frac{q}{C}$$

bateria $\rightarrow dW = -dq V \rightarrow$ ∈ pot elétrica

$dW = -dU$

$V = \frac{q}{C} \Rightarrow dU = \frac{q\,dq}{C} \Rightarrow U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2}$

$$\frac{U}{V_{ol}} = \frac{\varepsilon_0}{2}|\vec{E}|^2$$

07/04/10

$$\frac{dU}{dV_{ol}} = u = \frac{\varepsilon_0}{2} |\vec{E}|^2$$

$$U = \frac{1}{2} \int qV$$

$$U = \frac{1}{2} \int \rho(R) . V(R) \, dV_{ol}$$

$$\vec{\nabla} \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$U = \frac{\varepsilon_0}{2} \int V(R) \vec{\nabla} . \vec{E} \, dV_{ol}$$

$$V(R) \vec{\nabla} \vec{E} = \vec{\nabla}(V(R) \vec{E}) - \vec{E} . \vec{\nabla} f$$

Mas

$$\vec{\nabla} V = - \vec{E}$$

$$\Downarrow$$

$$U = \frac{\varepsilon_0}{2} \int \left[\vec{\nabla} (V \vec{E}) + |\vec{E}|^2 \right] dV_{ol}$$

$$U = \frac{\varepsilon_0}{2} \int \vec{\nabla} (V \vec{E}) \, dV_{ol} + \frac{\varepsilon_0}{2} \int |\vec{E}|^2 \, dV_{ol}$$

$$\hookrightarrow = \oint_S (V \vec{E}) . d\vec{S}$$

$$V \propto \frac{1}{R}$$

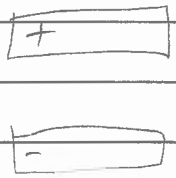
$$|\vec{E}| \propto \frac{1}{R^2} \qquad R \to \infty = \int V |\vec{E}| \, dS \to 0$$

$$dS \propto R^2$$

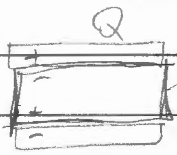
$$\boxed{U = \frac{\varepsilon_0}{2} \int |\vec{E}|^2 \, dV_{ol}}$$

credeal

Dielétricos



$$E = \frac{\sigma}{\varepsilon_0}$$



→ condutor

$E = 0$

Isolante

$$\int E\,dS = \frac{q_{int}}{\varepsilon_0} = \frac{Q - q'}{\varepsilon_0}$$

$$K = \frac{E_0}{E} = \frac{Q}{Q - q'} = \text{cte dielétrica}$$

↳ prop do mat

No Vácuo

$$E_0 = \frac{Q}{\varepsilon_0 A} \qquad \Delta V_0 = \frac{Qd}{\varepsilon_0 A} \qquad C_0 = \frac{\varepsilon_0 A}{d}$$

Dielétrico

$$E = \frac{Q}{K\varepsilon_0 A} \qquad \Delta V = \frac{Qd}{K\varepsilon_0 A} \qquad C = \frac{K\varepsilon_0 A}{d}$$

$$\varepsilon = K\varepsilon_0$$

↳ permissividade elétrica

$$-Ea + EKa = \frac{q_p}{\varepsilon_0}$$

$$\sigma = \frac{q_p}{a}$$

$$\sigma = \varepsilon_0 E (K - 1)$$

Momento de Dipolo

$d\vec{p} = dq \cdot d\,\hat{z} = \sigma_p\, dV_{ol}\, \hat{z}$

$\frac{d\vec{p}}{dV_{ol}} = \sigma_p \hat{z} = \vec{P}$ (polarização dielétrica)

$\vec{P} = \chi \cdot \varepsilon_0 \vec{E}$ p/ mtos dielétricos

χ = susceptibilidade dielétrica

$\vec{P} = \frac{d\vec{p}}{dV_{ol}} = \sigma_p \hat{z}$

$|\vec{P}| = \sigma_p = \chi \varepsilon_0 |\vec{E}| = \varepsilon_0 \cdot |\vec{E}| (K-1)$

$\chi = K-1 \in \vec{P} = \chi \varepsilon_0 \vec{E}$

Polarização Inomogênea

$\sigma_p = \vec{P} \cdot \hat{n}$

Fluxo:

$\int_S \hat{n} \cdot \vec{P}\, dA = \int_S \sigma_p\, dA = -Q_p$

$\boxed{\vec{\nabla} \cdot \vec{P} = -\rho_p}$

No Vácuo

$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

No dielétrico

$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{Livres} + \rho_{P\,(polariz)}}{\varepsilon_0}$

$\rho_p = -\vec{\nabla} \cdot \vec{P}$ $\quad \varepsilon_0 \vec{\nabla} \cdot \vec{E} = \rho_L - \varepsilon_0 \vec{\nabla}(\chi \vec{E})$

$\hookrightarrow \vec{\nabla} \left[(1+\chi) \vec{E} \right] = \frac{\rho_L}{\varepsilon_0} \Rightarrow K \vec{\nabla} \cdot \vec{E} = \frac{\rho_L}{\varepsilon_0}$

$\vec{D} = K\vec{E}$

↳ vet de desloc

$$\boxed{\vec{\nabla} \cdot \vec{D} = \frac{\rho_L}{\varepsilon_0}}$$

Cap 6 Corrente Elétrica

$$i = \frac{dq}{dt}$$

$\xleftarrow{\vec{i}}$

$e^{-} \; \vec{v} \rightarrow$

$$[i] = C/s = A$$

Densidade de corrente

$$\vec{j} = \frac{d\vec{i}}{dS} \Rightarrow di = \vec{j} \cdot d\vec{S}$$

$$\rho = \frac{dq}{dVol}$$

$$\frac{dq}{dt} = di = \rho \vec{v} \cdot d\vec{S}$$

$$\Rightarrow \vec{j} = \rho \vec{v}$$

Conservação de Carga e eq da continuidade

$$\oint \vec{j} \cdot d\vec{S} = -\frac{dq}{dt} = -\int \frac{\partial \rho}{\partial t} dVol$$

$$\int \vec{\nabla} \cdot \vec{j} \, dVol = -\int_{Vol} \frac{\partial \rho}{\partial t} dVol$$

$$\vec{\nabla} \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

Corrente estacionária:

$$\frac{\partial \rho}{\partial t} = 0 \Rightarrow \vec{\nabla} \cdot \vec{j} = 0$$

22/04/10

Lei de Ohm:

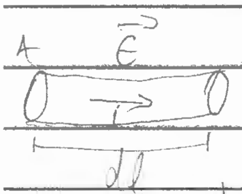
$V = Ri$

$\vec{P} = \varepsilon_0 \chi \vec{E}$

Caso particular

$\vec{j} = \sigma \vec{E}$

↳ condutibilidade elétrica → material



$\Delta V_{BA} = V_B - V_A$

$dV = V_A - V_B = \int_A^B \vec{E} \cdot d\vec{l} = E\,dl$

$i = \int \vec{j} \cdot d\vec{S}$

$i = jS = \sigma E . S$

$E = \frac{i}{\sigma S}$

$dV = \frac{i}{\sigma S} dl$

$dV = \underbrace{\frac{dl}{\sigma S}}\, i$

dR = resistência

$\rho = \frac{1}{\sigma}$ = resistividade

$dV = dR\, i$

P/ mtos materiais.

$\rho = \rho_0 [1 + \alpha (T - T_0)]$

$R = \frac{\rho l}{S}$

credeal

$j = ne\langle v\rangle$

$\langle v\rangle = \frac{j}{ne}$

Colisões $\Leftrightarrow$ Resistividade

$p = m_e \langle v\rangle$

$F = e\,\mathcal{E}$

$F\Delta t = \Delta p$

$m_e\langle v\rangle = e\,\mathcal{E}\Delta t$

$\langle v\rangle = \frac{e\,\mathcal{E}\Delta t}{m_e} \Rightarrow j = \frac{ne^2\,\mathcal{E}\Delta t}{m_e}$

$\sigma = \frac{ne^2\Delta t}{m_e}$ → entre colisões

Resistência ñ pode ser explicada por esse método
resultados do tamanho do Δt ñ batem:

$l_{contas} = 30\,\text{Å}$

$l_{quântica} = 2{,}6\,\text{Å}$

Efeito Joule

$dW = dq\,V = i\,dt\,V$

$$\boxed{P = \frac{dW}{dt} = iV = \frac{V^2}{R} = i^2R}$$

$\mathcal{E} = \frac{dV}{dl}$ $\quad dP = i\,dl\,\mathcal{E} = j\,S\,dl\,\mathcal{E} = j\,\mathcal{E}\,dVol$

$$\boxed{\frac{dP}{dVol} = \sigma\mathcal{E}^2}$$

27/04/10

Força Eletromotriz

$$\vec{j} = \sigma \vec{E}$$

$$\vec{E}\begin{cases} \vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \vec{\nabla}\times\vec{E} = 0 \end{cases}$$

i estacionária $\dfrac{\partial \rho}{\partial t} = 0$

$$\vec{\nabla}\cdot\vec{j} = -\frac{\partial \rho}{\partial t} \Rightarrow \vec{\nabla}\vec{j} = 0 \Rightarrow \vec{\nabla}\vec{E} = 0 \neq \frac{\rho}{\varepsilon_0}$$

$\Downarrow$

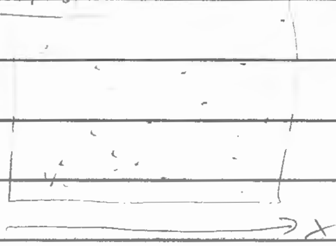
impossível

$\nexists$ i mantida somente por $\vec{E}$

$$\vec{j} = \sigma\left(\vec{E} + \vec{E}^e\right)$$

↳ campo ñ eletrostático

Pilha



difusão tende homogêneo

$$\vec{g} = -D\vec{\nabla}n$$

Dissociação de íons

H^+Cl^-
H^+Cl^-
H^+Cl^-
$\Rightarrow$ $-Q$ $+Q$

$$\vec{j}_{difusao} = -e\left(D_+ - D_-\right)\vec{\nabla}n$$

forças: $\vec{E}$ e atrito viscoso

$$\vec{v} = \mu \vec{F}$$

↳ mobilidade iônica

$$\vec{j}_\mathcal{E} = ne\left(\mu_+ F_+ - \mu_- F_-\right)$$

$$D_+ \gg D_- \Rightarrow \mu_+ \gg \mu_-$$

$$\vec{j} = \vec{j}_{dif} + \vec{j}_\mathcal{E}$$

$$\vec{j} \approx -eD_+ \vec{\nabla} n + ne\mu_+ \vec{F}_+$$

$$F = e\mathcal{E}$$

$$\vec{j} \approx eD_+ \vec{\nabla} n + ne^2 \mu_+ \mathcal{E}$$

$$\sigma \mathcal{E} = ne^2 \mu_+ \mathcal{E}$$

$$\boxed{\sigma = ne^2 \mu_+}$$

$$\sigma \mathcal{E}^e = -eD_+ \vec{\nabla} n$$

$$\vec{\mathcal{E}}^e = -\frac{D_+}{e\mu_+} \cdot \frac{\vec{\nabla} n}{n}$$ origem cinética do campo ñ eletrostático

$$\mathcal{E} = \oint_c (\vec{\mathcal{E}} + \vec{\mathcal{E}}^e)\, dr = \int_2^1 \vec{\mathcal{E}}^e \cdot d\vec{r}$$

Equilíbrio

$$\vec{j} = 0 \Rightarrow \vec{\mathcal{E}} = -\vec{\mathcal{E}}^e$$

$$\mathcal{E} = \int_2^1 \vec{\mathcal{E}}^e \cdot d\vec{r} = \int_1^2 \vec{\mathcal{E}} \cdot d\vec{r} = V_1 - V_2$$

$$V_+ > V_-$$

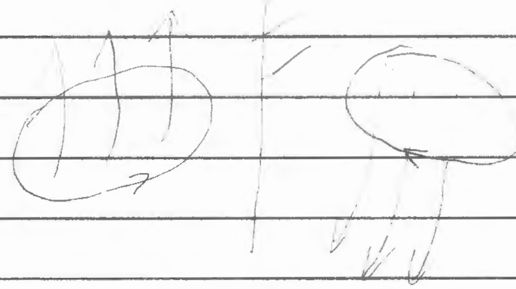
29/04/10

Campo Magnético

N S

$$\vec{F} = q\vec{v} \times B$$

$\vec{B} \rightarrow$ pseudo-vetor / vetor axial



$[B] = \text{Tesla} = T$

$1 \text{ Gauss} = 10^{-4}\, T$

$B_{Terra} = 0,6 \text{ Gauss}$

$$\vec{F}_{Lorentz} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$d\vec{l}$ $\quad dt \quad$ $\vec{v} = \dfrac{d\vec{l}}{dt}$

$q \quad \vec{v}$

$$dW = \vec{F} \cdot \vec{v}\, dt = q\vec{E} \cdot \vec{v}\, dt$$

$$\boxed{P = \frac{dW}{dt} = q\vec{E} \cdot \vec{v}}$$

$W = 0 \Rightarrow E_c = cte$

Geração e transmissão

Choques

$P_{dis} = R i^2$

$P_{trans} = V i$

$R = \frac{mV}{qB}$

$\phi_B = \oint_S \vec{B} \cdot d\vec{S} = 0$

$\oint_S \vec{B} \cdot d\vec{S} = \int_V \vec{\nabla} \cdot \vec{B} \, dV = 0 \Rightarrow \boxed{\vec{\nabla} \cdot \vec{B} = 0}$

↳ ñ ∃ monopolo

F_B ñ realiza trab

$\vec{j} = nq \langle \vec{v} \rangle$

$\vec{f} = \vec{j} \times \vec{B}$ → densidade de força

dl

A e^-

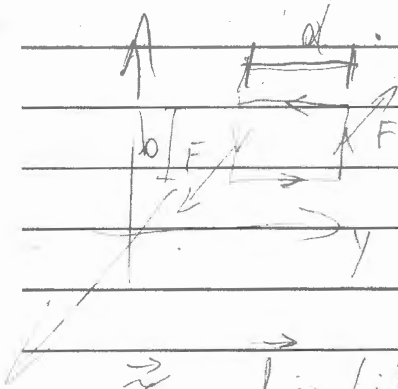
$d\vec{F} = \vec{j} \times \vec{B} \, A \, dl$

$\boxed{d\vec{F} = i \, d\vec{l} \times \vec{B}}$

$\vec{F} = i \oint_C d\vec{l} \times \vec{B}$

$\vec{B} = cte$

$\vec{F} = i \left(\oint_C d\vec{l} \right) \times \vec{B} = 0$



$\vec{B} = B\,\hat{y}$

$F = 0$, $\vec{\tau} = \vec{a} \times \vec{F}$

$\vec{\tau} = \vec{l} \times (i\vec{b} \times \vec{B})$

$\vec{\tau} = l(-\hat{y}) \times (ibB(-\hat{z}) \times \hat{y})$

$\vec{\tau} = ibaB(\hat{y}) \times (\hat{x}) = iabB\,\hat{z}$

$\vec{\tau} = iSB\,\hat{z}$

$\vec{\tau} = \underbrace{(i\vec{S})} \times \vec{B}$

$\vec{m} \rightarrow$ momento de dipolo magnético

Equilíbrio $\Rightarrow$ $\vec{m} \times \vec{B} = 0 \Rightarrow \vec{m} \parallel \vec{B} \Rightarrow \vec{B} \perp$ plano da espira

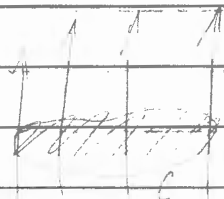
$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

corrente elétrica $\Rightarrow \vec{B}$

$\vec{\nabla} \cdot \vec{B} = 0$

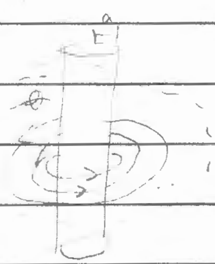
$\rightarrow$ permeabilidade magnética no vácuo

$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 i \rightarrow$ corrente total que atravessa a superfície definida por C



$i = \int_S \vec{J} \cdot d\vec{S}$

$\oint_C \vec{B} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{S} = \mu_0 \int_S \vec{J} \cdot d\vec{S}$



$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

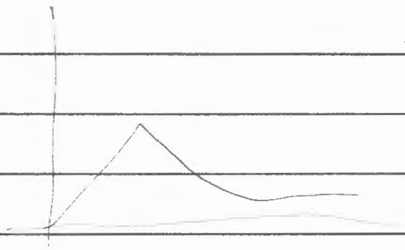
$B \oint_C dl = \mu_0 i$

$B = \frac{\mu_0 i}{2\pi r}$, $r > a$

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 \frac{i}{\pi a^2}\cdot \pi r^2$$

$$B \cdot 2\pi r = \frac{\mu_0 i r^2}{a^2}$$

$$B = \frac{\mu_0 i}{2\pi} \cdot \frac{r}{a^2}, \quad r < a$$



Briot - Savart

$d\vec{l}$ elemento de corrente $i d\vec{l}$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{l} \times \vec{r}}{r^3}$$ (Halliday)

$$dB = \frac{\mu_0 i}{4\pi} \cdot \frac{dl \operatorname{sen}\theta}{r^2}$$ corrente estacionária

$$\mu_0 = 4\pi \cdot 10^{-7} N/A$$

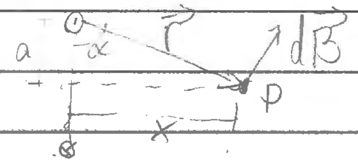
$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B}) = 0 = \mu_0 \vec{\nabla}\cdot\vec{j} + \vec{\nabla}\frac{\partial \vec{E}}{\partial t}\mu_0\varepsilon_0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{j} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla}\vec{j} = -\frac{\partial \rho}{\partial t}$$

12/09/10

Espira



$$B = \int dB \cos\alpha = \frac{\mu_0 i a}{4\pi r^2}\cos\alpha \int_0^{2\pi} d\varphi =$$

$$= \frac{\mu_0 i}{2r^2} \cdot \frac{a}{r} = \frac{\mu_0 i}{2} \cdot \frac{a^2}{r^3}$$

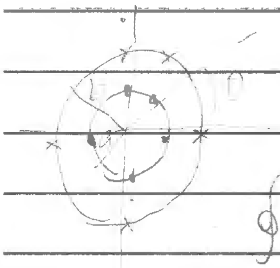
$$= \frac{\mu_0 i}{2} \cdot \frac{a^2}{(a^2+x^2)^{3/2}}$$

$x >> a$

$$\vec{B} = \frac{\mu_0 i}{2\pi} \cdot \frac{S}{x^3}\hat{x} = \frac{\mu_0}{2\pi} \frac{\vec{m}}{x^3}$$

$$\vec{E} = \frac{1}{2\pi\varepsilon_0} \cdot \frac{\vec{p}}{x^3}$$

Campo Magnético de um Toróide

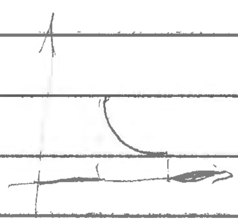


$$\vec{B} = B(r)\hat{\theta}$$

$$\oint \vec{B}\, d\vec{l} = 2\pi B r$$

$$\vec{B} = \frac{\mu_0 N i}{2\pi r}\hat{\theta}, \quad a < r \leq b$$

$$\vec{B} = \vec{0}, \quad r < a, \ r \geq b$$



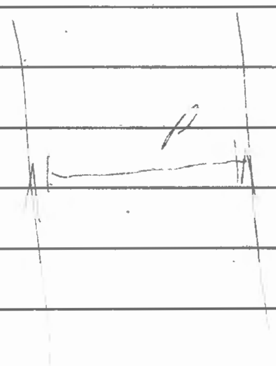
$$\vec{B} = \mu_0 i n \frac{R}{r}\hat{\theta}$$

$$R = \frac{1}{2}(a+b)$$

$$n = \frac{N}{2\pi R}$$

Solenóide

$$\left.\begin{matrix} R \to \infty \\ r \to \infty \end{matrix}\right\} \vec{B} = \mu_0 i n \hat{\theta}$$



$$\vec{B} = \frac{\mu_0 i}{2\pi\rho}\hat{\theta}$$

$$\vec{F}_{2(1)} = i_2 \, d\vec{l}_2 \times \vec{B}$$
$$= \frac{\mu_0 i_1 i_2}{2\pi\rho} \, dl \, \hat{z} \times \hat{\theta}$$

$$\frac{\vec{F}_{2(1)}}{dl_2} = -\frac{\mu_0 i_1 i_2}{2\pi\rho}\hat{\rho}$$

$A \rightarrow F = 1N$

$\rho = 1m$

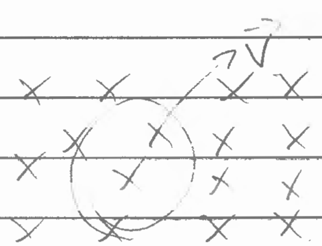
Faraday

Eletro $\Leftrightarrow$ Magnético

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

Experimento

Espira móvel num $\vec{B}$ cte



$$\vec{F} = -e\,\vec{v}\times\vec{B}$$

$$\vec{F} = -e\,\vec{E}^e$$

↳ ~~com~~ $\vec{E}$ equivalente

$$\vec{E}^e = \vec{v}\times\vec{B}$$

$$\mathcal{E} = \oint_C \vec{E}^e\cdot d\vec{\ell}$$

Surge $\mathcal{E}$ gerado p/ ~~o~~ um $\vec{E}^e$ derivado da $\vec{F}$ de Lorentz

$$\oint_S B\cdot dS = 0$$

$$\phi_B^{t} - \phi_B^{t+dt} = \int_{S_\ell} B\,dS_\ell = \int \vec{B}\cdot(d\vec{\ell}\times\vec{v}\,dt) = dt\int d\vec{\ell}\cdot(\vec{v}\times\vec{B})$$

$$= dt\int d\vec{\ell}\cdot\vec{E}^e = \mathcal{E}\,dt = -d\phi_B$$

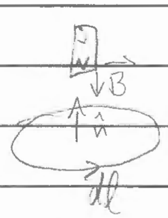
$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

Forma diferencial

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

Lei de Lenz



$\vec{B}\cdot\hat{n} < 0 \Rightarrow \phi < 0$

imã se aproxima $\Rightarrow |\vec{B}|\uparrow$ e $|\phi|\uparrow \Rightarrow \frac{d|\phi|}{dt} > 0$

↳ $\frac{d\phi}{dt} < 0$

$\oint E\, dl = -\frac{d\phi}{dt} > 0 \Rightarrow \vec{E}\,\vec{dl} > 0$

$\vec{E} \,//\, \vec{dl}$

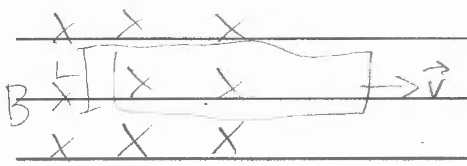
$\mathcal{E} = \oint \vec{E}\cdot\vec{dl} = Ri$

$i // dl$

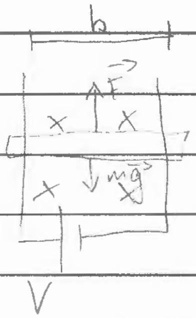
$\vec{E}_{ind} = -\vec{B}$?

↳ se opõe ao fluxo

↳ é cte



$\mathcal{E} = BLv$



equilíbrio

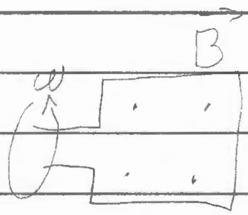
$F = mg$, $v = cte$

$\frac{V - Bhv}{R}\, hB = mg$

R = resistência

$v = \frac{1}{hB}\left(V - \underbrace{\frac{mgR}{hB}}_{i^*}\right)$

$V = i^* R + Bhv$

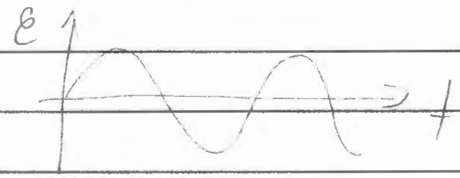


n espiras

$\theta_{B,dS} = \omega t$

$$\phi = \int \vec{B} \cdot d\vec{S} = nBS\cos(\omega t)$$

$$\mathcal{E} = -\frac{d\phi}{dt} = nBS\omega \,\mathrm{sen}(\omega t)$$



$$\mathcal{E} = Ri \Rightarrow i = \frac{nBS\omega \,\mathrm{sen}(\omega t)}{R}$$

$$\vec{m} = niS\hat{n}$$

$$\tau = |\vec{m} \times \vec{B}| = iSnB\,\mathrm{sen}(\omega t)$$

$$P = \omega \tau = iSnB\omega\,\mathrm{sen}(\omega t)$$

Indutância

N_1

$R_1 < R_2$

$$B_1 = \mu_0 \frac{N_1}{l} i_1 \quad , \quad r \leq R_1$$

$$\phi_{2(1)} = N_2 \int \vec{B}_1 \cdot d\vec{S} = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l} i_1$$

produzido por

$$L_{2(1)} = \frac{\mu_0 N_1 N_2 \pi R_1^2}{l} = L_{1(2)}$$

$\hookrightarrow$ indutância mútua

$\phi_{1(2)} = L i_2$

$\phi_{2(1)} = L i_1$

~~$\phi_{1(1)}$~~

$$L_{1(1)} = \frac{\mu_0 N_1^2}{l} \pi R_1^2$$

$$L_{2(2)} = \frac{\mu_0 N_2^2}{l} \pi R_2^2$$

$$L = \frac{\mu_0 N^2}{l} \pi R^2$$

↳ auto-indutância

$$\phi_1 = L_1 i_1 + L_{12} i_2$$

$$\phi_2 = L_2 i_2 + L_{21} i_1$$

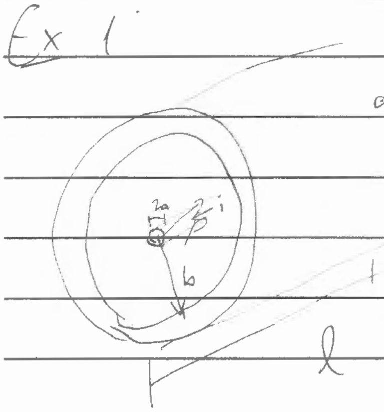
Generalizando

$$\phi = L i$$

$$\mathcal{E} = -\frac{d\phi}{dt} = -L\frac{di}{dt}$$

Ex 1

$a \leq \rho \leq b$



$$\phi = \int \vec{B} \cdot d\vec{S} = i$$

$$= l \int_a^b \frac{\mu_0 i}{2\pi \rho} d\rho$$

$$L = \frac{\mu_0 l}{2\pi} \ln \frac{b}{a}$$

09/06/10

Energia Magnética

$p = \frac{dW}{dt}$ $\mathcal{E} = -L\frac{di}{dt}$

$p = -\mathcal{E} i$

$p = L\frac{di}{dt}i = \frac{dW}{dt}$

$U = \int_0^t \frac{dW}{dt}dt = \int_0^t L i\,di$

$$\boxed{U = \frac{Li^2}{2}}$$

Capacitor

$U_T = U_B + U_E$

$U_T = \frac{Li^2}{2} + \frac{Q^2}{2C}$ $\frac{U_T}{Vol} = \frac{B^2}{2\mu_0} + \frac{\varepsilon_0 E^2}{2}$

$\frac{U_B}{Vol} = \frac{B^2}{2\mu_0}$

Cap 10

Circuitos

1. Elementos idealizados

(A) Resistor

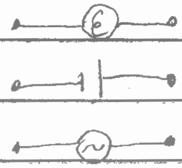
$V = RI$, $P_{dis} = RI^2$

(B) Capacitor $V = \frac{Q}{C}$, $U = \frac{CV^2}{2} = \frac{Q^2}{2C}$

(C) Indutor

$V = L\frac{dI}{dt}$, $U = \frac{LI^2}{2}$

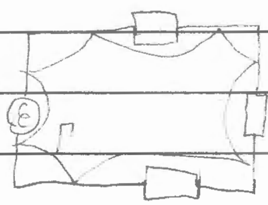
① Gerador



→ contínua $V = \varepsilon$

alternada

② Leis de Kirchoff



$$\oint_{\Pi} \vec{E} \cdot d\vec{l} = 0 = -\frac{d\phi_B}{dt}$$

$$\vec{\nabla} \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

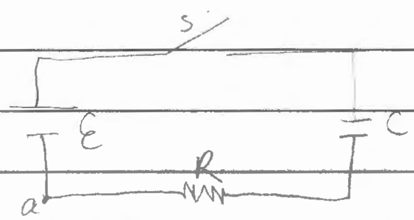
$\sum V = 0$ em um circuito

↳ 1ª Lei

Lei da continuidade ($\sum i = cte$)

↳ 2ª Lei

③ Circuito RC com fonte de tensão contínua



$$\varepsilon = \frac{Q}{C} + Ri$$

$$\boxed{R\frac{dQ}{dt} + \frac{Q}{C} = \varepsilon}$$

EDO

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = 0$$

$\lambda^2 + a\lambda + b = 0$

$\Delta > 0 \Rightarrow x(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$

$\Delta = 0 \Rightarrow x(t) = (C_1 + tC_2)e^{\lambda t}$

$\Delta < 0 \Rightarrow x(t) = Ce^{\mathrm{Re}(\lambda)t}\cos(\mathrm{Im}(\lambda)t + \delta)$

09/06/10

$$\frac{d^2x}{dt^2} + a\frac{dx}{dt} + bx = S(t)$$

$$x = x_H + x_{NH}$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = \mathcal{E}$$

$$Q_H = e^{-\frac{t}{RC}} \qquad \boxed{Q = K_1 e^{-\frac{t}{RC}} + \mathcal{E}C}$$

$$Q_{NH} = \mathcal{E}C$$

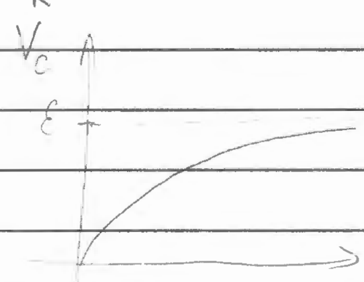
$$Q(0) = 0 \Rightarrow K_1 = -\mathcal{E}C$$

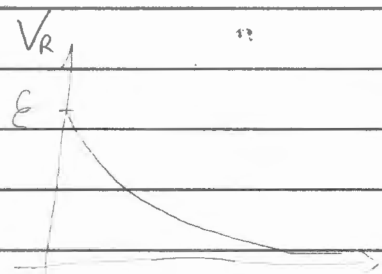
$$Q(t) = C\mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}}$$

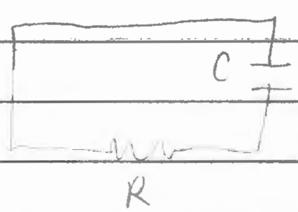
$$V_C = \mathcal{E}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V_R = \mathcal{E} e^{-\frac{t}{RC}}$$





4) RC sem fonte de tensão



$$Ri + \frac{Q}{C} = 0$$

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \Rightarrow Q(t) = K_1 e^{-\frac{t}{RC}}$$

$$Q(0) = Q_0 = C\mathcal{E} = K_1$$

$$Q(t) = C\mathcal{E} e^{-\frac{t}{RC}}$$

credeal

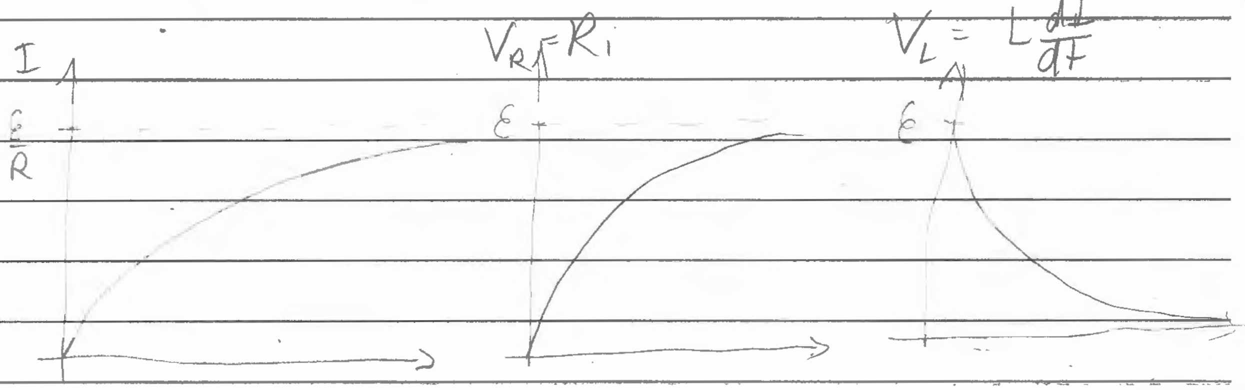
5 RL com fonte de tensão contínua

$$RI + L\frac{dI}{dt} = \mathcal{E}$$

$$I_H = K_1 e^{-\frac{tL}{R}} \qquad I_p = \frac{\mathcal{E}}{R} \quad \Rightarrow \quad I(t) = \frac{\mathcal{E}}{R} + K_1 e^{-\frac{tL}{R}}$$

$$I(0) = 0$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\frac{tL}{R}}\right)$$



LC

$$\frac{d^2I}{dt^2} + \frac{I}{LC} = 0 \qquad \omega_0 = \sqrt{\frac{1}{LC}}$$

$$I(t) = A\cos\left(\frac{t}{LC} + \delta\right)$$

$$Q(0) = Q_0 \qquad I(0) = I_0 \quad \Rightarrow \quad \operatorname{tg}\delta = \frac{Q_0\omega_0}{I_0}$$

$$A = \sqrt{I_0^2 + Q_0^2\omega_0^2}$$

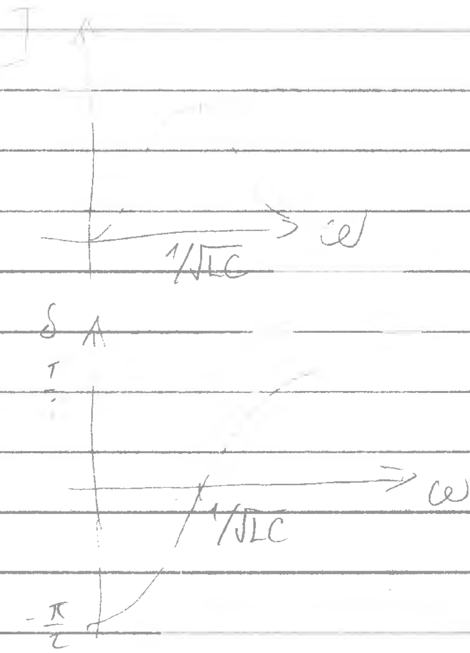
$$I(t) = A\cos(\omega_0 t + \delta)$$

$$Q(t) = \frac{A}{\omega_0}\operatorname{sen}(\omega_0 t + \delta)$$

09/06/10

Caso de estudo

$I_0 = 0 \Rightarrow \begin{cases} A = Q_0 \omega_0 \\ \delta = \frac{\pi}{2} \end{cases}$



$$I(t) = -\omega_0 Q_0 \,\text{sen}(\omega_0 t)$$
$$Q(t) = Q_0 \cos(\omega_0 t)$$

Energia

$$U_C(t) = \frac{Q^2}{2C}$$

$$U_C(t) = \frac{Q_0^2 \cos^2(\omega_0 t)}{2C}$$

$$U_L = \frac{1}{2} L I^2 = \frac{1}{2} L \omega_0^2 Q_0^2 \,\text{sen}^2(\omega_0 t)$$

$$U_L = \frac{Q_0^2}{2C} \,\text{sen}^2(\omega_0 t)$$

$$U_T = \frac{Q_0^2}{2C} = c^{te}$$

Massa-Mola	LC
$m \frac{d^2x}{dt^2} + kx = 0$	$L \frac{d^2 I}{dt^2} + \frac{I}{C} = 0$
$x, m, k, \omega_0 = \sqrt{\frac{k}{m}}$	$I, L, \frac{1}{C}, \omega_0 = \sqrt{\frac{1}{LC}}$
$E_c = \frac{1}{2} m v^2$	$E_m = \frac{1}{2} L I^2$
$E_e = \frac{1}{2} k x^2$	$E_c = \frac{Q^2}{2C}$

09/06/10

⑥ RLC sem fonte

$$\frac{d^2I}{dt^2} + \gamma\frac{dI}{dt} + \omega_0^2 I = 0$$

ⓐ $\frac{\gamma}{2} > \omega_0$ Supercrítico

ⓑ $\frac{\gamma}{2} = \omega_0$ crítico

ⓒ $\frac{\gamma}{2} < \omega_0$ subcrítico

$$\lambda = -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$$

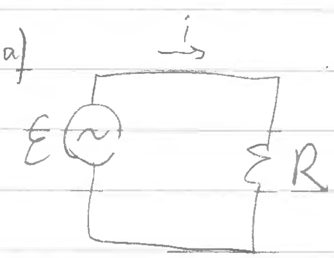
$\gamma \ll \omega_0 \Rightarrow \omega_1 = \omega_0$

$$I(t) = A e^{-\frac{\gamma}{2}t}\cos(\omega_0 t + \varphi)$$

$$Q(t) = \frac{A}{\omega_0} e^{-\frac{\gamma}{2}t}\,\mathrm{sen}(\omega_0 t \pm \varphi)$$

Corrente Alternada

a)



$\mathcal{E} = \mathcal{E}_m \,\mathrm{sen}(\omega t)$

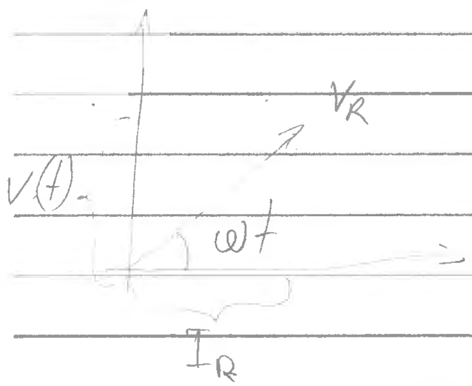
$\mathcal{E} = V(t) = Ri$

$V_R = \mathcal{E}_m$

$I_R = \frac{V_R}{R}$

(26/06/10)

Fasor



Aplicações

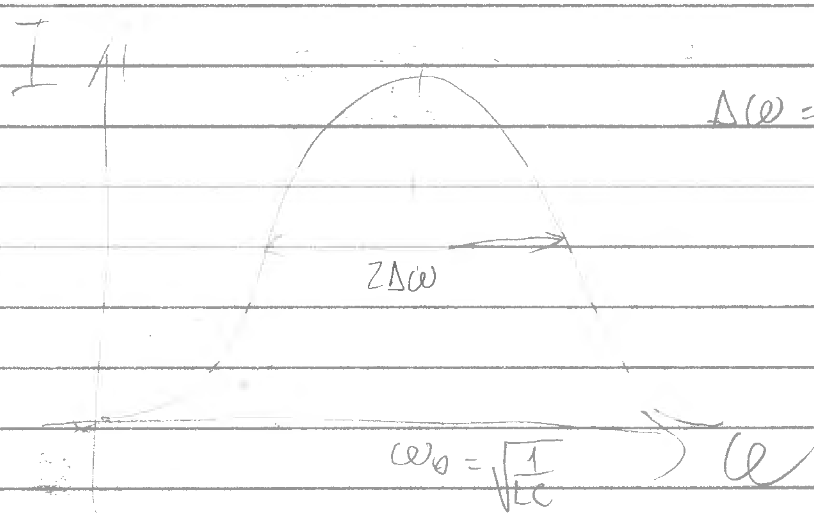
① $\omega \approx \frac{1}{\sqrt{LC}}$

$$I = \frac{\varepsilon_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= R\sqrt{1 + \left(\frac{\omega_0 L}{R}\right)^2 \left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^2}$$

$$Q = \frac{\omega_0 L}{R} \Rightarrow I = \frac{\varepsilon_m / R}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}}$$

~~PRA~~ $\omega = \omega_0 \Rightarrow \boxed{I = \varepsilon_m / R} \Rightarrow$ Ressonância



$$\Delta\omega = \frac{1}{\sqrt{2}} \cdot \frac{\varepsilon_m}{R}$$

16/06/10

Potência

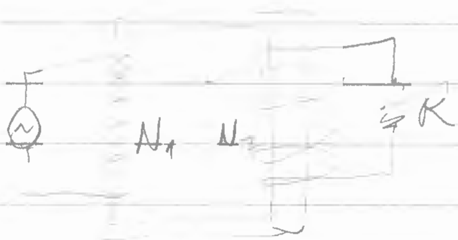
$P = R i^2$

$P = I^2 R \operatorname{sen}^2(\omega t - \phi)$ $\qquad \overline{\operatorname{sen}^2\theta} = \frac{1}{2}$

$\bar{P} = \frac{1}{2} R I^2 = R\left(\frac{I}{\sqrt{2}}\right)^2$

$I_{RMS} = \frac{I}{\sqrt{2}}$

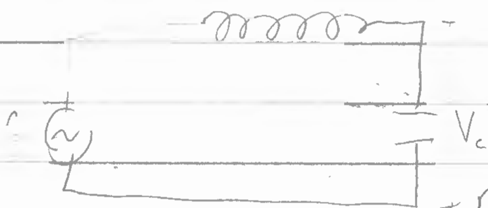
Transformador



$\varepsilon_1 = N_1 \frac{d\phi}{dt} \Rightarrow \frac{d\phi}{dt} = \frac{\varepsilon_1}{N_1} = \frac{\varepsilon_2}{N_2}$

Filtros

(1)



$\varepsilon = I z \Rightarrow I = \frac{\varepsilon}{z}$

$z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$R = 0 \Rightarrow z = \omega L - \frac{1}{\omega C}$

impedância

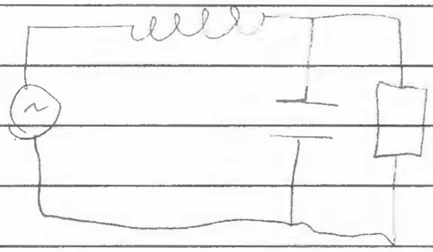
$V_c = I X_c \quad , \quad X_c = \frac{1}{\omega C}$

$\omega_0 = \sqrt{\frac{1}{LC}} \Rightarrow V_c = \frac{\varepsilon}{\frac{\omega^2}{\omega_0^2} - 1}$

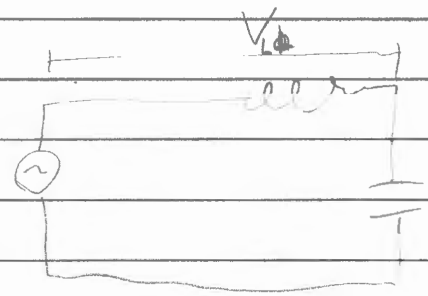
$\omega = 0 \Rightarrow V_c = -\varepsilon \Rightarrow$ passa baixas frequencias

17/06/10

$\omega \to \infty \Rightarrow V_C \to 0 \Rightarrow$ barra altas freqs



(2)



$\mathcal{E} = IZ$, $Z = \omega L - \frac{1}{\omega C}$

$V_L = I X_C$, $X_C = \omega L$

$$V_L = \frac{\mathcal{E}}{1 - \frac{\omega_0^2}{\omega^2}}$$

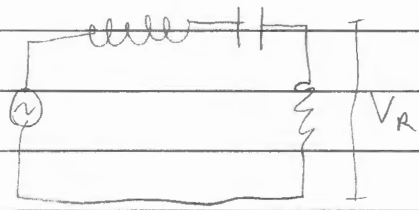
$\omega \to 0 \Rightarrow V_L \to 0$

barra baixas freqs

$\omega \to \infty \Rightarrow V_L \to \mathcal{E}$

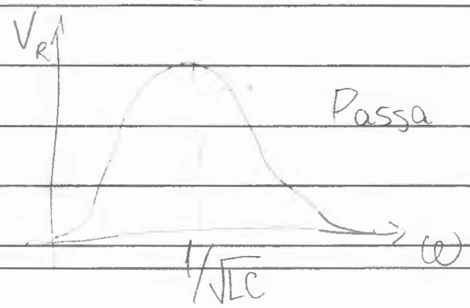
passa altas freqs

(3)



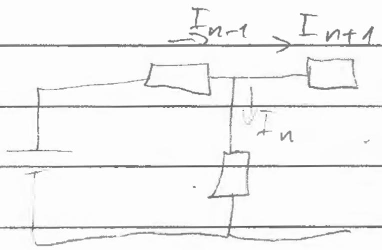
$V_R = RI$

$$V_R = \frac{\mathcal{E} R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$



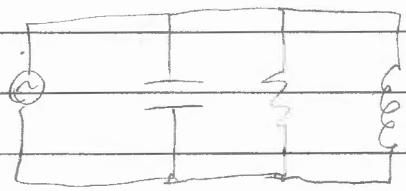
Passa banda

④ Malhas



$$I_{n+1} - \left(2 + \frac{Z_1}{Z_2}\right) I_n + I_{n-1} = 0$$

$$I_n = I_0 e^{\alpha n} \Rightarrow \alpha \Rightarrow I_n$$



$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{X_L} + \frac{1}{X_C} = \frac{1}{R} - \frac{i}{\omega L} + i\omega C$$

$$I = \frac{\mathcal{E}}{Z_{eq}}$$