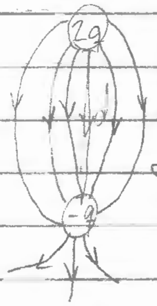
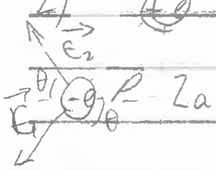
Cap 3

1)



A qtdde de linhas de força que saem de 2q ~~e~~ em direção a -q é o dobro do saldo de -q, já que a magnitude tem o dobro

2)



(+e) i) $\vec{E} = \vec{E}_1 + \vec{E}_2$

$\vec{E} = 2|\vec{E}_1| \cos\theta\, \hat{\rho}$

$\vec{E} = 2 \frac{1}{4\pi\epsilon_0} \cdot \frac{e}{a^2+\rho^2} \cdot \frac{\rho}{(a^2+\rho^2)^{1/2}} \hat{\rho}$

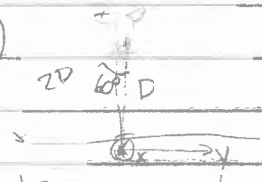
(+e) $\vec{E} = \frac{e\rho}{2\pi\epsilon_0 (a^2+\rho^2)^{3/2}} \hat{\rho}$

ii) $m\omega^2 \rho = -e\vec{E} = -\frac{e^2 \rho}{2\pi\epsilon_0 (a^2+\rho^2)^{3/2}}$

$\omega^2 = -\frac{e^2}{2\pi m \epsilon_0 (a^2+\rho^2)^{3/2}}$

$E \cdot 2\cancel{\pi r^2} = \frac{\cancel{\pi r^2}\sigma}{\epsilon_0}$

3)



$dE = \frac{k\,dQ}{D^2+x^2+y^2} = \frac{k\sigma\,dx\,dy}{D^2+x^2+y^2}$

$E = k\sigma \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dx\,dy}{D^2+x^2+y^2} \frac{D}{\sqrt{D^2+x^2+y^2}} = k\sigma \int_0^{\infty}\int_0^{2\pi} \frac{D\rho\,d\theta\,d\rho}{(D^2+\rho^2)^{3/2}} = 2\pi k\sigma D \int_0^{\infty} \frac{\rho\,d\rho}{(D^2+\rho^2)^{3/2}} =$

$dQ = \sigma\,dx\,dy$

$= 2\pi k\sigma D \int_0^{\pi/2} \frac{D\,tg\varphi\, D\sec^2\varphi\, d\varphi}{D^3 \sec^3\varphi} = -2k\sigma\pi \cos\varphi \Big|_0^{\pi/2} = \frac{\sigma}{2\epsilon_0}$

$\frac{\rho}{D} = tg\,\varphi$

$d\rho = D\sec^2\varphi\, d\varphi$

$E_s = 2\pi k\sigma \int_0^{\pi/3} \operatorname{sen}\varphi\, d\varphi = -\frac{\sigma}{2\epsilon_0} \cos\varphi \Big|_0^{\pi/3} = \frac{\sigma}{4\epsilon_0}$

$u = \cos\varphi$

$du = -\operatorname{sen}\varphi\, d\varphi$

$\frac{E_s}{E} = \frac{1}{2}$ cqd

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4) $\lambda = \frac{Q}{l}$ a)

$Q = l\lambda$

$\lambda = \frac{dQ}{d\rho}$

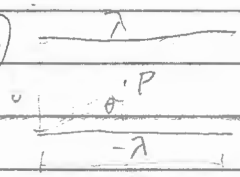
$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{\rho^2}\hat{x} = \frac{\lambda d\rho}{4\pi\varepsilon_0\rho^2}\hat{x}$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_d^{d+l}\frac{d\rho}{\rho^2}\hat{x}$$

$$\vec{E} = -\frac{\lambda}{4\pi\varepsilon_0}\left.\frac{1}{\rho}\right|_d^{d+l}\hat{x}$$

$$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{d} - \frac{1}{d+l}\right)\hat{x} = \frac{\lambda l}{4\pi\varepsilon_0 d(d+l)}\hat{x}$$

b) $E = \frac{9.10^9 . 3.10^{-6}}{5.10^{-2}.10.10^{-2}} = 5,4.10^{6}\ N/C$

5) 

$$d\vec{E} = -\frac{2dQk}{\left(\frac{b}{2}\right)^2 + x^2}\cos\theta\,\hat{z} = -\frac{2k\lambda dx}{\left(\frac{b}{2}\right)^2 + x^2}\cdot\cos\theta\,\hat{z} =$$

$$= -\frac{2k\lambda\frac{b}{2}\sec^2\theta d\theta}{\left[\left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}tg\theta\right)^2\right]}\cos\theta\,\hat{z} =$$

$\lambda = \frac{dQ}{dx}$

$\frac{2x}{b} = tg\,\theta$

$x = \frac{b}{2}tg\theta$

$dx = \frac{b}{2}\sec^2\theta d\theta$

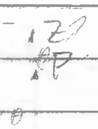
$\alpha = arctg\frac{a}{b}$

$$= -\frac{2k\lambda\frac{b}{2}\sec^2\theta}{\frac{b^2}{2}\sec^2\theta}d\theta\,\hat{z} = -\frac{4k\lambda}{b}\cos\theta\, d\theta\,\hat{z}$$

$$\vec{E} = -\int_{-arcsen\frac{a}{\sqrt{a^2+b^2}}}^{arcsen\frac{a}{\sqrt{a^2+b^2}}}\frac{4k\lambda}{b}\cos\theta\, d\theta\,\hat{z}$$

$$\vec{E} = -\frac{8k\lambda a}{b\sqrt{a^2+b^2}}\hat{z} = -\frac{2\lambda a}{\pi\varepsilon_0 b\sqrt{a^2+b^2}}$$

resp. menor

6) 

Pela simetria: $|E_x| = |E_y| = 0$

$E_z \to$ Será calculado apenas para um lado e seu módulo multiplicado por 4, já q o sentido é o msm.

dQ

$2l$

$\lambda = \frac{dQ}{dx}$

$$\frac{dE}{4} = \frac{dQ\cos\theta}{4\pi\varepsilon_0(D^2+l^2+x^2)} = \frac{\lambda dx . D}{\pi\varepsilon_0(D^2+l^2+x^2)^{3/2}} = \frac{\lambda D\sqrt{D^2+l^2}\sec^2\varphi\, d\varphi}{\pi\varepsilon_0(D^2+l^2)^{3/2}\sec^3\varphi}$$

$\lambda = \frac{Q}{2l}$

$x = \sqrt{D^2+l^2}\,tg\,\varphi$

$dx = \sqrt{D^2+l^2}\sec^2\varphi\, d\varphi$

$$E = \frac{\lambda D}{\pi\varepsilon_0(D^2+l^2)}\int_{-arcsen\frac{l}{\sqrt{D^2+2l^2}}}^{arcsen\frac{l}{\sqrt{D^2+2l^2}}}\cos\varphi\, d\varphi = \frac{2\lambda D l}{\pi\varepsilon_0(D^2+l^2)\sqrt{2l^2+D^2}}$$

$l \Rightarrow tg\,\varphi = \frac{l}{\sqrt{D^2+l^2}}$ $\quad\frac{l^2}{D^2+l^2}$ $\quad 1 - \frac{1}{\cos^2\varphi}$ $\quad\cos\varphi = \frac{\sqrt{D^2+l^2}}{\sqrt{2l^2+D^2}}$ $\quad sen\,\varphi = \frac{l}{\sqrt{2l^2+D^2}}$

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7) $\phi_E = \frac{q_{int}}{\varepsilon_0}$

a) $\phi_E = \frac{q}{6\varepsilon_0}$

b) $\phi_E = \frac{q}{24\varepsilon_0}$ (opostas)

$\phi_E = 0$ (adjacentes)

8) $E_1 = 300\,N/C$

$\vec{E}_1 = -300\hat{z}\ N/C$

$h = 1400\,m \Rightarrow E = 20\hat{z}\ N/C$

~~$\rho = q = ?$~~

$\Delta E A = \frac{\rho A h}{\varepsilon_0}$ $\quad 280 = \frac{\rho \cdot 1400}{\varepsilon_0}$

$\rho = \frac{\varepsilon_0}{5}$

$\rho = \frac{1}{20\pi k} = 1{,}8 \cdot 10^{-12}\ C/m^3$

9) $\vec{E}_1 \uparrow \quad \sigma \quad \downarrow \vec{E}_2$ $\quad |\vec{E}_1| = |\vec{E}_2|$

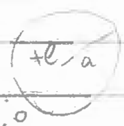
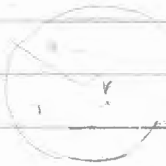
1

$\vec{E}_1 \downarrow \quad \uparrow \vec{E}_2 \Rightarrow \vec{E}_{acima} = \vec{0}$

2

$\vec{E}_1 \downarrow \quad -\sigma \quad \uparrow \vec{E}_2 \Rightarrow \vec{E}_{entre} = -2 \cdot \frac{\sigma}{2\varepsilon_0}\hat{z} = -\frac{\sigma}{\varepsilon_0}\hat{z}$

$\Rightarrow \vec{E}_{abaixo} = \vec{0}$

10)  a) 

$+e, a$

$a = 1\,\mathring{A}$

$\oint \vec{E} \cdot \hat{n}\, dS = \frac{\frac{4\pi r^3}{4\pi a^3} e}{\varepsilon_0}$

$E \oint_S dS = \frac{e r^3}{a^3 \varepsilon_0} = E \cdot 4\pi r^2$ $\quad \vec{E} = \frac{e\vec{r}}{4\pi\varepsilon_0 a^3}$

b) $F = -eE = -\frac{e^2 r}{4\pi\varepsilon_0 a^3} \Rightarrow MHS$ $\quad E \cdot 4\pi r^2 = \frac{e}{\varepsilon_0}$

c) $\omega^2 = \frac{e^2}{4\pi\varepsilon_0 a^3 m}$

$\omega = \frac{e}{2a\sqrt{\pi\varepsilon_0 a m}}$

$f = \frac{e}{4\pi a\sqrt{\pi\varepsilon_0 a m}} = 2{,}5 \cdot 10^{15}\ Hz$

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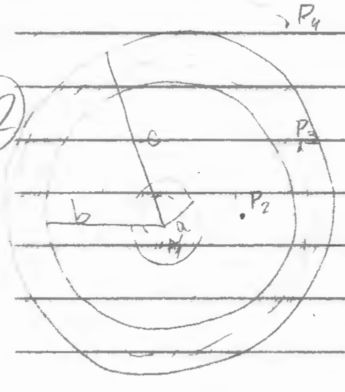
(11) $div(c \times r) =$

(12)



$$\oint_S \vec{E}\cdot\hat{n}\,dS = \frac{q_{int}}{\varepsilon_0}$$

a) $$E \cdot 4\pi r_1^2 = \frac{\rho \frac{4}{3}\pi r_1^3}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho r_1}{3\varepsilon_0}\hat{r}$$

b) $$E \cdot 4\pi r_2^2 = \frac{\rho \frac{4}{3}\pi a^3}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho a^3}{3\varepsilon_0 r_2^2}\hat{r}$$

c) $$E \cdot 4\pi r_3^2 = \frac{\rho\left(\frac{4}{3}\pi a^3 + \frac{4}{3}\pi r_3^3 - \frac{4}{3}\pi b^3\right)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho(a^3 - b^3 + r_3^3)}{3\varepsilon_0 r_3^2}\hat{r}$$

d) $$E \cdot 4\pi r_4^2 = \frac{\rho\left(\frac{4}{3}\pi a^3 + \frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3\right)}{\varepsilon_0}$$

$$\vec{E} = \frac{\rho(a^3 - b^3 + c^3)}{3\varepsilon_0 r_4}\hat{r}$$

(13) $\rho(r) = \rho_0 e^{-\frac{r}{a}}$

a) $dQ = \rho(r)\,dV = \rho_0 e^{-\frac{r}{a}}(r\,d\theta \quad r\,sen\,\varphi\,d\varphi \,.\, dr)$

$$Q = \int_0^\infty\int_0^\pi\int_0^{2\pi} r^2\rho_0 e^{-\frac{r}{a}} sen\theta\, d\theta\, d\varphi\, dr = 2\pi\rho_0\int_0^\infty\int_0^\pi r^2 e^{-\frac{r}{a}} sen\varphi\, d\varphi\, dr = 4\pi\rho_0\int_0^\infty r^2 e^{-\frac{r}{a}}dr =$$

$$= 4\pi\rho_0\left(-r^2 a e^{-\frac{r}{a}} + \int_0^\infty 2rae^{-\frac{r}{a}}dr\right)\Big|_0^\infty = 4\rho_0\pi\left(-r^2ae^{-\frac{r}{a}} + 2a\left(-are^{-\frac{r}{a}} + \int ae^{-\frac{r}{a}}dr\right)\right)\Big|_0^\infty =$$

$u = r^2 \quad du = 2r\,dr \qquad u = r \quad du = dr$

$dv = e^{-\frac{r}{a}}dr \quad v = -ae^{-\frac{r}{a}} \qquad dv = e^{-\frac{r}{a}} \quad v = -ae^{-\frac{r}{a}}$

$$= 4\pi\rho_0\left(-r^2ae^{-\frac{r}{a}} - 2a^2re^{-\frac{r}{a}} - 2a^3e^{-\frac{r}{a}}\right)\Big|_0^\infty =$$

$$= -4\pi\rho_0 a e^{-\frac{r}{a}}(r^2 + 2ar + 2a^2)\Big|_0^\infty =$$

$= 4\pi\rho_0 a \cdot 2a^2 = 8\pi\rho_0 a^3$

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b) $\oint_S \vec{E}\cdot\hat{n}\,dS = \frac{q_{int}}{\varepsilon_0}$

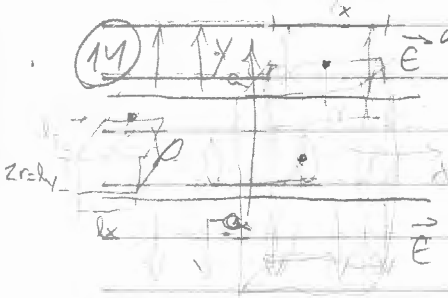
$E\cdot 4\pi r^2 = -\frac{4\pi\rho_0 a e^{-\frac{r}{a}}}{\varepsilon_0}\left(r^2+2ar+2a^2\right)\Big|_0^r$

$Er^2 = \frac{2\rho_0 a^3}{\varepsilon_0} - \frac{\rho_0 a e^{-\frac{r}{a}}}{\varepsilon_0}(r^2+2ar+2a^2)$

$E = \frac{2\rho_0 a^3}{\varepsilon_0 r^2} - \frac{\rho_0 a e^{-\frac{r}{a}}}{\varepsilon_0}\left(1+\frac{2a}{r}+\frac{2a^2}{r^2}\right)$

$E = \frac{2\rho_0 a^3}{\varepsilon_0 r^2}\left[1-\frac{1}{2}e^{-\frac{r}{a}}\left(\frac{r^2}{a^2}+\frac{2r}{a}+2\right)\right]$

(14)



$\oint_S \vec{E}\cdot\hat{n}\,dS = \frac{q}{\varepsilon_0}$

$E\,2l_xl_z = \frac{2a l_x l_z \rho}{\varepsilon_0}$

$\vec{E} = \frac{a\rho}{\varepsilon_0}\hat{z} = \vec{E}_{cima} = \vec{E}_{baixo}$

$E\cdot 2l_xl_z = \frac{\rho l_z l_x\cdot 2r}{\varepsilon_0}$

$\vec{E}_{meio} = \frac{\rho z}{\varepsilon_0}\hat{z}$

b) $\operatorname{div} E = \frac{\rho}{\varepsilon_0}$

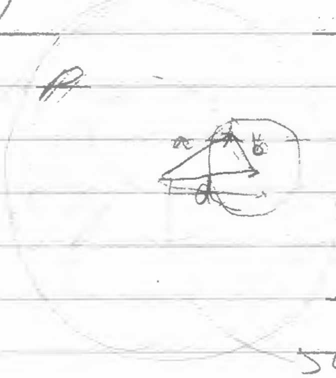
Cima e baixo:

$\operatorname{div} E = 0 = \frac{\rho}{\varepsilon_0}$ ✓

Meio:

$\operatorname{div} E = \frac{\rho}{\varepsilon_0}$ ✓

(15)



$\vec{E} = \vec{E}_{esf} - \vec{E}_b$

$E_{esf}\cdot 4\pi a^2 = \frac{\frac{4}{3}\pi a^3\rho}{\varepsilon_0} = \frac{\vec{a}\rho}{3\varepsilon_0}$

$\vec{E} = \frac{\vec{a}\rho}{3\varepsilon_0} - \frac{\vec{b}\rho}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}(\vec{a}-\vec{b})$

$\vec{E}_b\cdot 4\pi b^2 = \frac{\frac{4}{3}\pi b^3\rho}{\varepsilon_0} = \frac{\vec{b}\rho}{3\varepsilon_0}$

$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{d} = \vec{cte}$

$\vec{d} = \vec{a}-\vec{b}$

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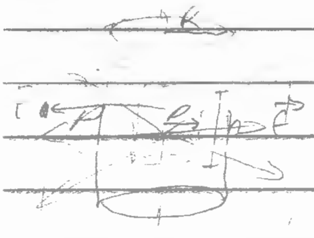
16) interno

$$E \oint_S dS = \frac{q_{int}}{\varepsilon_0}$$



$$E\, 2\pi\rho h = \frac{\pi\rho^2 h\delta}{\varepsilon_0}$$

$$E = \frac{\delta}{2\varepsilon_0}\vec{\rho}$$

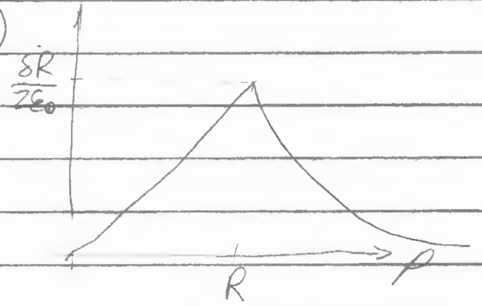
externo:

$$E\, 2\pi\rho h = \frac{\pi R^2 h\delta}{\varepsilon_0}$$

$$\vec{E} = \frac{\delta R^2}{2\rho\varepsilon_0}\hat{\rho}$$

b) $0<\rho<R \Rightarrow |\vec{E}| = \frac{\delta\rho}{2\varepsilon_0}$

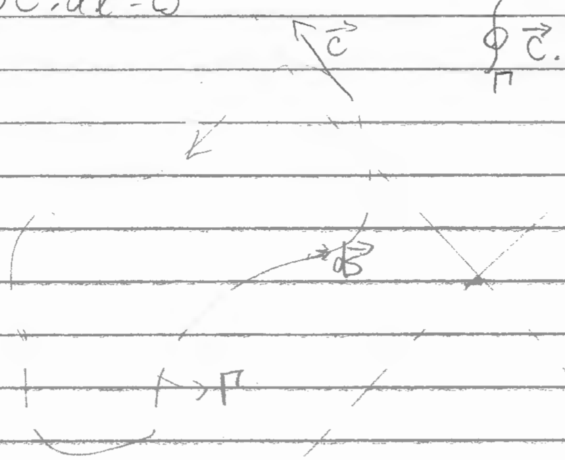
d)



C

$$\oint \vec{E}\cdot\vec{dl} = 0$$

$$\oint_\Gamma \vec{C}\cdot\vec{dS}$$



$$\oint_\Gamma \vec{C}\cdot\vec{dS} = \int_A (\vec{\nabla}\times\vec{C})\cdot\hat{n}\,dA$$

$$\vec{\nabla}\times\vec{E} = 0$$

DATAPEL