28/03/10

Cap 5

(1) $R = 2\mu m$

$d = 5cm$

$\rho = 0,78 g/cm^3$

$mg = bv$

$\uparrow V = 40kV \Rightarrow v_f = 2 v$

$|mg(-\hat{z})| + b.2v(\hat{z}) + F_{el} = \vec{0}$

$F_{el} = (mg - 2bv)\hat{z} = -bv\hat{z} = bv(-\hat{z}) \Rightarrow$ carga negativa

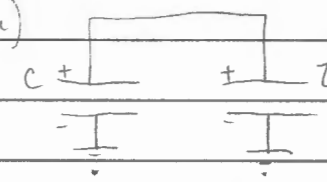
$q\mathcal{E} = -bv$ $\quad q = -\frac{5.10^{-2} \cdot 780 \cdot \frac{4}{3}\pi (2.10^{-6})^3 \cdot 98}{4.10^{4}}$

$q\frac{V}{d} = -mg$

$q = -\frac{dmg}{V}$ $\quad q = -3,2.10^{-19} C = -2e$

(2) $C_1 = C$ $\quad C_2 = 2C$ $\quad Q_1 = Q_2 = Q$

a)



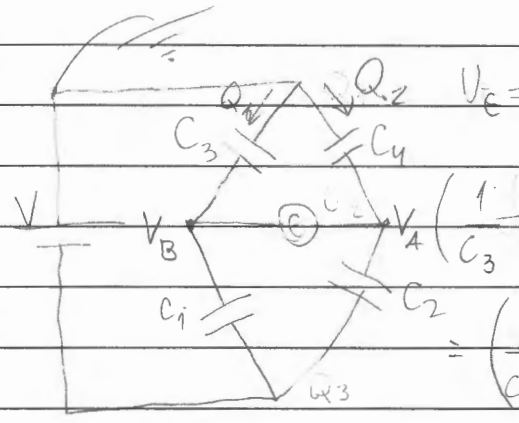

$C_{eq} = 3C$

$V = \frac{Q_{eq}}{C_{eq}} = \frac{2Q}{3C}$

b) $\Delta U = U_0 - (U_1 + U_2) = \frac{2Q \cdot \frac{2Q}{3C}}{2} - \left(\frac{Q \cdot \frac{Q}{C}}{2} + \frac{Q \cdot \frac{Q}{2C}}{2}\right) = \frac{2Q^2}{3C} - \frac{Q^2}{2C} - \frac{Q^2}{4C} = -\frac{Q^2}{12C}$

c) É convertida em luz, calor, etc...

3)




$$\left(\frac{1}{C_3}+\frac{1}{C_2}\right)^{-1} + \left(\frac{1}{C_1}+\frac{1}{C_4}\right)^{-1} = \left(\frac{1}{C_3}+\frac{1}{C_1}\right)^{-1} + \left(\frac{1}{C_4}+\frac{1}{C_2}\right)^{-1} =$$

$$= \left(\frac{1}{C_3+C_4} + \frac{1}{C_1+C_2}\right)^{-1}$$

$$Q_1 = \left(\frac{1}{C_1}+\frac{1}{C_3}\right)^{-1} V$$

$$Q_2 = \left(\frac{1}{C_2}+\frac{1}{C_4}\right)^{-1} V$$

$$V_A = -\frac{Q_2}{C_4} = -\frac{C_2 C_4 V}{C_4(C_2+C_4)}$$

$$V_B = \frac{Q_1}{C_3} + \frac{C_4 C_3 V}{C_3(C_1+C_3)}$$

$$V_A = V_B$$

$$\frac{C_2 V}{C_2+C_4} = \frac{C_1 V}{C_1+C_3}$$

$$C_4 C_1 + C_4 C_3 + C_2 C_1 + C_2 C_3 = C_1 C_2 + C_1 C_4 + C_3 C_2 + C_3 C_4$$

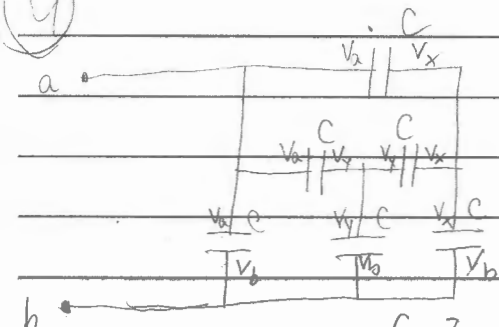
$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

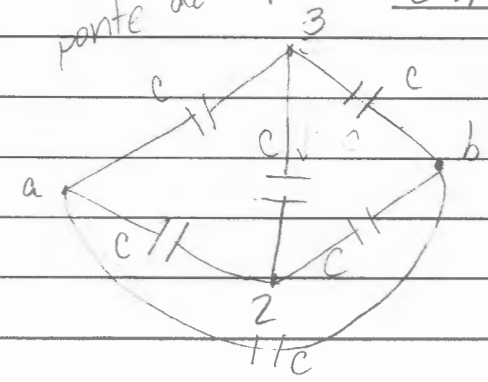
Ч.Т.Д.

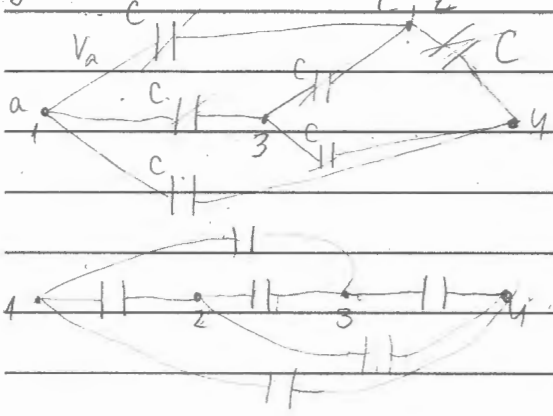
09/04/10

punte de capacitanțe

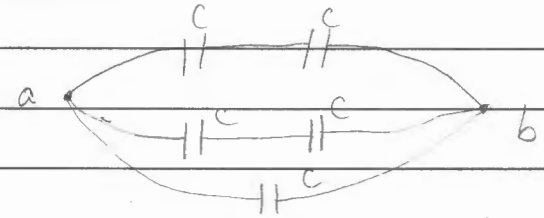
4)



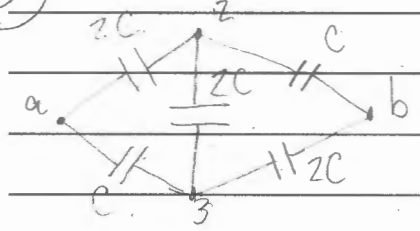




$$\frac{C_1}{C_2} = \frac{C_3}{C_4} \Rightarrow V_2 = V_3$$




$$C_{eq} = \left(\frac{1}{c}+\frac{1}{c}\right)^{-1} + \left(\frac{1}{c}+\frac{1}{c}\right)^{-1} + c = 2C$$

5)



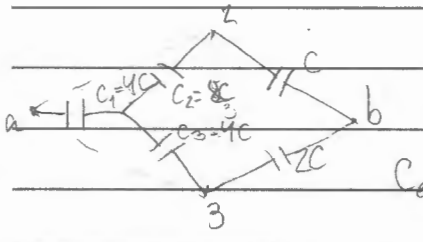

$$\frac{1}{C_{a2}} = \frac{3}{8C} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \frac{3}{8C} = \frac{2}{8C} + \frac{1}{C_2}$$

$$C_2 = 8C$$

$$\frac{1}{C_{a3}} = \frac{4}{8C} = \frac{1}{C_1} + \frac{1}{C_3} \qquad \frac{4}{8C} = \frac{2}{8C} + \frac{1}{C_3}$$

$$\frac{1}{C_{23}} = \frac{3}{8C} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_3 = 4C$$



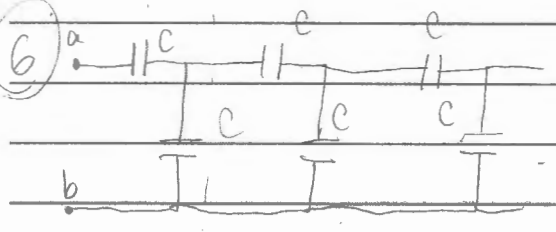

$$\frac{2}{C_1} + \frac{3}{8C} = \frac{3}{8C} + \frac{4}{28C}$$

$$C_1 = 4C$$

$$C_{eq} = \left\{\left[\left(\frac{1}{4C}+\frac{1}{2C}\right)^{-1} + \left(\frac{1}{8C}+\frac{1}{C}\right)^{-1}\right]^{-1} + \frac{1}{4C}\right\}^{-1} = \left[\left(\frac{4C}{3}+\frac{8C}{9}\right)^{-1} + \frac{1}{4C}\right]^{-1} =$$

$$= \left(\frac{9}{20C}+\frac{5}{20C}\right)^{-1} = \frac{10C}{7}$$

6)




$$\frac{1}{C} + \frac{1}{C+x} = \frac{1}{x} \qquad x = C_{eq}$$

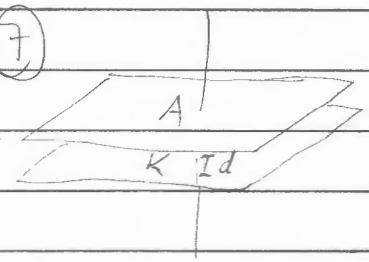
$$x(x+2C) = C(x+C) \quad \cdot x$$

$$x^2 + Cx - C^2 = 0$$

$$\Delta = C^2 + 4C^2 = (C\sqrt{5})^2 + 6C^2 + 1$$

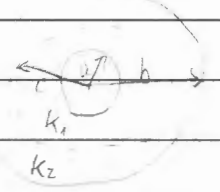
$$x = \frac{-C \pm C\sqrt{5}}{2} \Rightarrow C_{eq} = \frac{C}{2}(\sqrt{5}-1)$$

(7)




(8)

$$C = 4\pi\varepsilon_0 \left(\frac{R_1 R_2}{R_2 - R_1}\right)$$




$$C_1 = 4\pi k_1 \varepsilon_0 \left(\frac{ac}{c-a}\right)$$

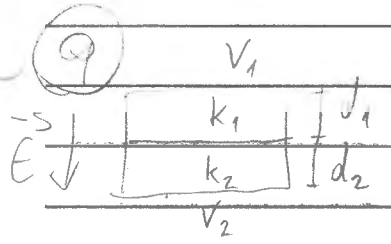
$$C_2 = 4\pi k_2 \varepsilon_0 \left(\frac{bc}{b-c}\right)$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4\pi k_1 \varepsilon_0}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{4\pi k_2 \varepsilon_0}\left(\frac{1}{c} - \frac{1}{b}\right) =$$

$$\frac{1}{C} = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{k_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{k_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]$$

$$C = 4\pi\varepsilon_0 \left[\frac{1}{k_1}\left(\frac{1}{a} - \frac{1}{c}\right) + \frac{1}{k_2}\left(\frac{1}{c} - \frac{1}{b}\right)\right]^{-1}$$

(9)




a) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$ $\qquad C_i = \frac{k_i \varepsilon_0 A}{d}$

$$\frac{1}{C} = \frac{d_1}{k_1 \varepsilon_0 A} + \frac{d_2}{k_2 \varepsilon_0 A}$$

$$C = \varepsilon_0 A \left( \frac{d_1}{k_1} + \frac{d_2}{k_2} \right)^{-1}$$

$$C = \varepsilon_0 A \left( \frac{k_1 k_2}{k_2 d_1 + k_1 d_2} \right)$$

b) $\sigma = ? = \frac{Q}{A}$

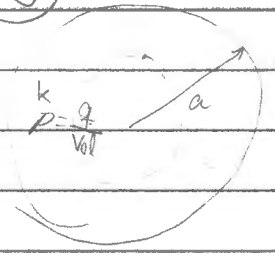
$$C = \frac{Q}{V} \Rightarrow Q = CV$$

$$\sigma = \frac{CV}{A} = \frac{\varepsilon_0 A V}{A} \left( \frac{k_1 k_2}{k_1 d_2 + k_2 d_1} \right)$$

$$\sigma = \varepsilon_0 V \left( \frac{k_1 k_2}{k_1 d_2 + k_2 d_1} \right)$$

(10)

a) $\vec{E} = ?$




$0 \le r \le a$

$$\int E \, ds = \frac{\frac{4}{3}\pi r^3 \rho}{k \varepsilon_0}$$

$$E_< \cdot \frac{4}{3}\pi r^2 = \frac{\frac{4}{3}\pi r^3 \rho}{3 k \varepsilon_0} \Rightarrow \vec{E}_< = \frac{\rho}{3 k \varepsilon_0} \vec{r}$$

$r > a$

$$\int E_> \, ds = \frac{\frac{4}{3}\pi R^3 \rho}{k \varepsilon_0} \qquad \vec{E}_> = \frac{\rho}{3 k \varepsilon_0} \cdot \frac{R^3}{r^2}$$

$$E_> \cdot 4\pi r^2 = \frac{4\pi R^3 \rho}{3 k \varepsilon_0}$$

b) $V = -\int \vec{E} \, d\vec{l} + \int \vec{E} \, d\vec{l} = \int \vec{E} \, d\vec{l}$

$$V = V(0) - V(a) = -\int_a^0 \frac{\rho r}{3 k \varepsilon_0} \hat{r} \cdot d\vec{l}$$

$$V = \int_0^a \frac{\rho r}{3 k \varepsilon_0} dr$$

$$V = \frac{\rho a^2}{6 k \varepsilon_0}$$