17/05/10

Cap 8

1 a) $L = \hbar = 1055.10^{-34}\ J.s$

$m\omega^2 R = F_{el}$

$m\omega^2 a_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{a_0^2}$

$m\omega^2 a_0^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{a_0}$

$I\omega^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{e^2}{a_0}$

$\rightarrow I^2\omega^2 = \frac{m a_0}{4\pi\epsilon_0} \cdot e^2$

$\hbar^2 = \frac{(me^2)\, a_0}{4\pi\epsilon_0}$

$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{me^2}$

$a_0 = \frac{(1055.\cancel{10^{-34}})^2 \cdot 10^{-8}}{9.10^9 . 9{,}1.10^{-31}(1{,}6.10^{-19})^2} =$

$= \frac{1{,}055^2}{81{,}9.2{,}56} . 10^{-8} = 5{,}3.10^{-3} . 10^{-8} = 0{,}53.10^{-10}$

$a_0 = 0{,}53\ Å$

b) $i = \frac{q}{t} = \frac{e\omega}{2\pi} = \frac{e\hbar}{2\pi I} = \frac{e\hbar}{2\pi m a_0^2} = \frac{1{,}6.10^{-19} . 1{,}055.10^{-34}}{2.3{,}14.9{,}1.10^{-31}. 0{,}53^2 . 10^{-20}} = 0{,}105.10^{-2}\ A$

$\doteq 1{,}05.10^{-3} A$

c) $dB = \frac{\mu_0 i}{4\pi} \frac{dl\, sen\theta}{r^2}$

$B = \frac{\mu_0 i}{4\pi a_0} \int dl = \frac{\mu_0 i}{4\pi a_0^2} . 2\pi a_0 \Rightarrow B = \frac{\mu_0 i}{2a_0}$

$B = \frac{4\pi.10^{-7} . 1{,}05.10^{-3}}{2.\ 0{,}53 . 10^{-10}} = 4\pi = 12{,}5\ T$

d) $\mu_B = iS = \frac{e\hbar}{2\pi m a_0^2} . \pi a_0^2 = \frac{e\hbar}{2m} = \frac{eL}{2m}$

$\frac{\mu_B}{L} = \frac{e}{2m}$

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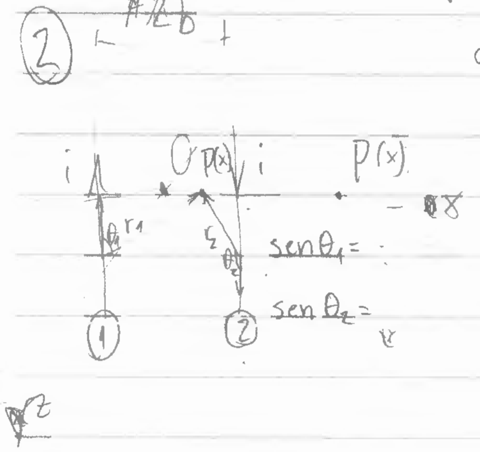
(2)



a) $|x| < b$

$$\oint B_1 \cdot dl = \mu_0 i$$

$$B_1\, 2\pi (x+b) = \mu_0 i$$

$$\vec{B}_1 = \frac{\mu_0 i}{2\pi(x+b)} (-\hat{z})$$

$$\vec{B}_2 = \frac{\mu_0 i}{2\pi(b-x)} (\hat{z})$$

$$\vec{B} = -\frac{\mu_0 i}{2\pi}\left(\frac{\cancel{x}+b+b-\cancel{x}}{b^2-x^2}\right)\hat{z}$$

$$\vec{B} = \frac{\mu_0 i b}{\pi(x^2-b^2)}\hat{z}$$

b) $|x| > b$

$$\oint B_1\, dl = \mu_0 i \qquad \oint B_2\, dl = \mu_0 i$$

$$B_1\, 2\pi(x+b) = \mu_0 i \qquad B_2\, 2\pi(x-b) = \mu_0 i$$

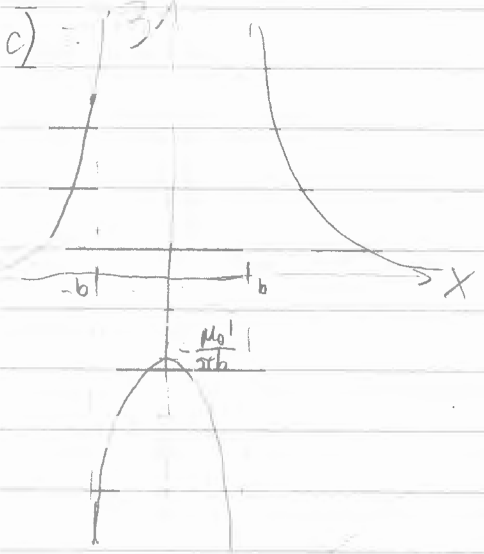
$$\vec{B}_1 = \frac{\mu_0 i}{2\pi(x+b)}(-\hat{z}) \qquad \vec{B}_2 = \frac{\mu_0 i}{2\pi(b-x)}(-\hat{z})$$

$$\vec{B} = -\frac{\mu_0 i}{2\pi}\left(\frac{\cancel{x}+\cancel{b}+b-\cancel{x}}{b^2-x^2}\right)\hat{z}$$

$$\vec{B} = \frac{\mu_0 i b}{\pi(x^2-b^2)}\hat{z}$$

$\vec{B} = B\hat{z}$

c)



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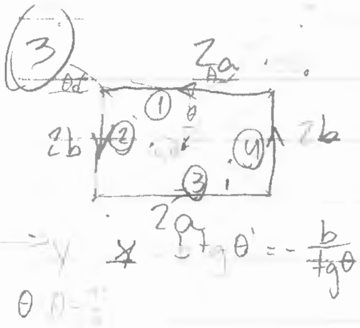
(3)



a) $d\vec{B}_{1,3} = \frac{\mu_0 i}{4\pi} \cdot \frac{dl\, sen\theta}{r^2}$

$tg\,\theta' = -\frac{b}{tg\theta} = -\frac{b\cos\theta}{sen\theta}$

$dy = \frac{b(sen^2\theta + cos^2\theta)}{sen^2\theta} = b\,cossec^2\theta\, d\theta$

$= \frac{\mu_0 i}{4\pi} \cdot \frac{dy\, sen\theta}{r^2} = \frac{\mu_0 i}{4\pi} \cdot \frac{b\,cossec^2\theta}{b^2} sen^2\theta\, d\theta$

$\vec{B}_1 = -\frac{\mu_0 i}{4\pi b} \int_{arccos\frac{a}{\sqrt{a^2+b^2}}}^{arccos\frac{-a}{\sqrt{a^2+b^2}}} sen\theta\, d\theta$

$\vec{B}_1 = \frac{\mu_0 i}{2\pi b} \frac{a}{\sqrt{a^2+b^2}}\,\hat{x}$

$\vec{B}_{1+3} = 2\vec{B}_{1,3}$

(Análogo)

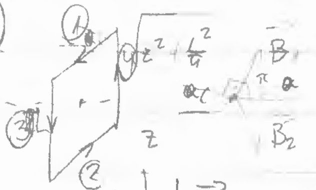
$\vec{B}_{2+4} = 2\vec{B}_{2,4}$

$a \gg b$

$\vec{B} = \frac{\mu_0 i}{\pi b}$

Problema 2 p/ $x = 0$

(4)



As componentes x e y se anulam

$d\vec{B}_{1z} = d\vec{B}_{2z} = d\vec{B}_{3z} = d\vec{B}_{4z} = d\vec{B}_z$

$dl = \frac{L}{2} cossec^2\theta d\theta$

$d\vec{B}_z = 4\frac{\mu_0 i}{4\pi} \cdot \frac{dl\, sen\theta}{r^2} = \frac{4\mu_0 i}{4\pi} \frac{\sqrt{\frac{L^2}{4}+z^2}\, cossec^2\theta d\theta \cdot sen\theta}{(\frac{L^2}{4}+z^2)/sen^2\theta} \frac{L/2}{\sqrt{\frac{L^2}{4}+z^2}}\,\hat{z}$

$d\vec{B}_z = \frac{4\mu_0 i}{4\pi} \cdot \frac{L/2}{\frac{L^2}{4}+z^2}\, sen\theta d\theta\, \hat{z}$

$\vec{B}_z = \frac{\mu_0 i}{2\pi} \frac{L}{L^2/4 + z^2} \left[-\cos\theta\right]_{arccos\frac{L/2}{\sqrt{\frac{L^2}{4}+z^2+\frac{L^2}{4}}}}^{-arccos\frac{L/2}{\sqrt{L^2/4+z^2+L^2/4}}} =$

$\vec{B} = \vec{B}_z = \frac{\mu_0 i}{2\pi} \cdot \frac{L^2}{(\frac{L^2}{4}+z^2)} \cdot \frac{1}{\sqrt{\frac{L^2}{2}+z^2}}\,\hat{z}$

$z = 0 \Rightarrow \vec{B} = \frac{\mu_0 i}{2\pi} \cdot 4 \cdot \frac{\sqrt{2}}{L}\hat{z} = \frac{2\mu_0 i\sqrt{2}}{\pi L}\hat{z}$

No ex 3 se $a = b = \frac{L}{2} \Rightarrow |\vec{B}| = \frac{4\mu_0 i}{\pi L^2} \cdot \frac{L}{\sqrt{2}} = \frac{2\mu_0 i\sqrt{2}}{\pi L}$

$z \gg L$

$\vec{B} = \frac{\mu_0 i}{2\pi} \cdot \frac{L^2}{z^3}\hat{z}$

$\vec{B} = \frac{\mu_0 \vec{m}}{2\pi z^3}$ $\quad \vec{m} = i L^2 \hat{z}$

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5 a)

1, 3: $i\,d\vec{l} \times \hat{r} = 0 \Rightarrow B_1 = B_3 = 0$

2: $d\vec{B} = \frac{\mu_0 i}{4\pi}\cdot\frac{d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0 i}{4\pi R^2}\cdot dl\,\hat{x}$

$$\vec{B} = \frac{\mu_0 i}{4\cancel{\pi} R^{\cancel{2}}}\cdot \cancel{\pi R}\,\hat{x} \Rightarrow \vec{B} = \frac{\mu_0 i}{4R}\hat{x}$$

b)

$$\vec{B}_1 = \vec{B}_2 = \frac{1}{2}\cdot\frac{\mu_0 i}{2\pi R}(-\hat{x}) \Rightarrow \vec{B}_{1+2} = \frac{\mu_0 i}{2\pi R}(-\hat{x})$$

ex (a) : $\vec{B}_3 = \frac{\mu_0 i}{4R}(-\hat{x}) \Rightarrow \vec{B} = \vec{B}_{1+2} + \vec{B}_3 = -\frac{\mu_0 i}{2R}\hat{x}\left(\frac{1}{\pi} + \frac{1}{2}\right)$

$$\vec{B} = -\frac{\mu_0 i\,(2+\pi)}{4\pi R}\hat{x}$$

$$B_f = I\left(\frac{\pi}{2}\right) - I\left(-\frac{\pi}{2}\right) = \underbrace{\left[I\left(\frac{\pi}{2}\right) - I(0)\right]}_{B_1} - \underbrace{\left[I\left(-\frac{\pi}{2}\right) - I(0)\right]}_{-B_1}$$

$B_{inf} = 2B_1$

$B_1 = \frac{1}{2}B_{inf}$

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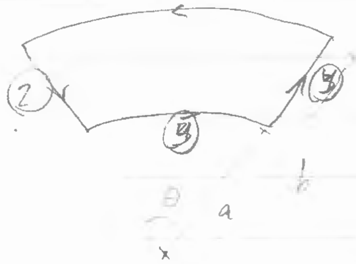
6



$$dB = \frac{\mu_0}{4\pi}\, i\frac{\vec{dl}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\, i\frac{dl\,\mathrm{sen}\,\theta_{dl,r}}{r^2}$$

$$①: dB_1 = \frac{\mu_0 i}{4\pi b^2}\, dl \Rightarrow B_1 = \frac{\mu_0 i}{4\pi b^2}\cdot \cancel{b}\theta$$

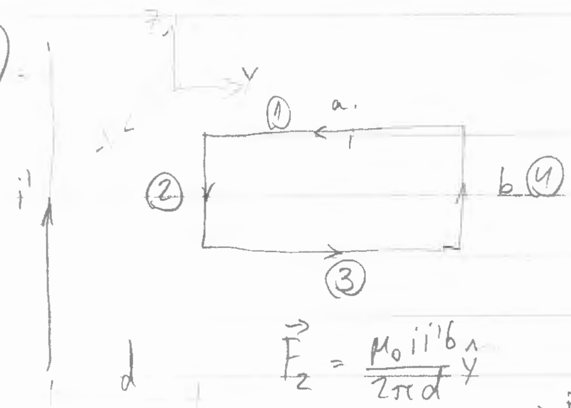
$$B_1 = \frac{\mu_0 i\theta}{4\pi b}\hat{z}$$

$$②: dB_2 = \frac{\mu_0 i}{4\pi}\cdot\frac{\vec{dl}\times\hat{r}}{r^2} \Rightarrow B_2 = 0$$

$$③: dB_3 = \frac{\mu_0 i}{4\pi a^2}\cdot dl \Rightarrow B_3 = \frac{\mu_0 i}{4\pi a^2}\cdot a\theta \Rightarrow B_3 = -\frac{\mu_0 i\theta}{4\pi a}\hat{z}$$

$$④: dB_3 = \frac{\mu_0 i}{4\pi}\cdot\frac{\vec{dl}\times\hat{r}}{r^2} \Rightarrow B_4 = 0 \Rightarrow B = B_1 + B_3 = \frac{\mu_0 i\theta}{4\pi}\left(\frac{1}{b}-\frac{1}{a}\right)\hat{z}$$

7



As componentes da força exercidas por ① e ③ se anulam ($①: (-\hat{z})$, $③: \hat{z}$)

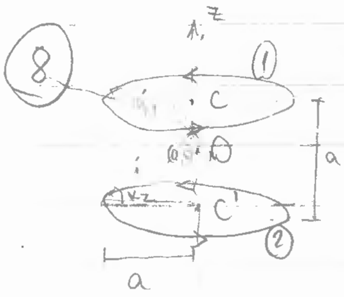
$$\vec{F}_2 = \frac{\mu_0 i i' b}{2\pi d}\hat{y}$$

$$\vec{F}_4 = -\frac{\mu_0 i i' b}{2\pi(d+a)}\hat{y}$$

$$\Rightarrow \vec{F} = \vec{F}_2 + \vec{F}_4 = \frac{\mu_0 i i'}{2\pi}\left(\frac{1}{d}-\frac{1}{d+a}\right)\hat{y}$$

$$\vec{F} = \frac{\mu_0 i i' a b}{2\pi d(d+a)}\hat{y}$$

8



$|z| < \frac{a}{2}$

$$d\vec{B}(z) = \left[\frac{\mu_0 N i}{4\pi}\cdot\frac{dl\cos\alpha_1}{(\frac{a}{2}-z)^2+a^2} + \frac{\mu_0 N i}{4\pi}\cdot\frac{dl\cos\alpha_2}{(\frac{a}{2}+z)^2+a^2}\right]\hat{z}$$

$$\vec{B}(z) = \frac{\mu_0 N i}{4\pi^2}\cdot 2\pi a^2\left(\frac{1}{[(\frac{a}{2}-z)^2+a^2]^{3/2}} + \frac{1}{[(\frac{a}{2}+z)^2+a^2]^{3/2}}\right)\hat{z}$$

$$\vec{B}(z) = \frac{\mu_0 N i\, a^2}{2}\cdot\left(\frac{1}{[(\frac{a}{2}+z)^2+a^2]^{3/2}} + \frac{1}{[(\frac{a}{2}-z)^2+a^2]^{3/2}}\right)\hat{z}$$

$$\vec{B}(0) = \frac{\mu_0 i a^2}{2}\left(\frac{4}{5a^2}\right)^{3/2} 2\hat{z} \Rightarrow \vec{B}(0) = \frac{\mu_0 i N}{a}\cdot\left(\frac{4}{5}\right)^{3/2}\hat{z}$$

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$$\vec{B}(z) = \frac{\mu_0 N i a^2}{2}\left(\frac{1}{\left[\left(\frac{a}{2}+z\right)^2+a^2\right]^{3/2}} + \frac{1}{\left[\left(\frac{a}{2}-z\right)^2+a^2\right]^{3/2}}\right)\hat{z}$$

$$\frac{d\vec{B}}{dz}(z) = -\frac{\mu_0 N i a^2}{2}\,\frac{3}{2}\left(\frac{2\left(\frac{a}{2}+z\right)}{\left[\left(\frac{a}{2}+z\right)^2+a^2\right]^{5/2}} - \frac{2\left(\frac{a}{2}-z\right)}{\left[\left(\frac{a}{2}-z\right)^2+a^2\right]^{5/2}}\right)\hat{z}$$

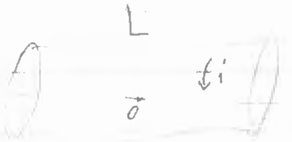
$$z=0 \Rightarrow \frac{d\vec{B}}{dz}(0) = -\frac{3\mu_0 i N}{2a^3}\cdot\left(\frac{a/2}{(4/5)^{5/2}} - \frac{a/2}{(4/5)^{5/2}}\right)\hat{z} = 0$$

$$\frac{d^2\vec{B}}{dz^2}(z) = -\frac{\mu_0 N i a^2}{2}\left(\frac{\left[\left(\frac{a}{2}+z\right)^2+a^2\right]^{5/2} - \left(\frac{a}{2}+z\right)\frac{5}{2}\left[\left(\frac{a}{2}+z\right)^2+a^2\right]^{3/2}2\left(\frac{a}{2}+z\right)}{\left[\left(\frac{a}{2}+z\right)^2+a^2\right]^{5}} - \right.$$

$$\left. - \frac{\left[\left(\frac{a}{2}-z\right)^2+a^2\right]^{5/2} + \left(\frac{a}{2}-z\right)\frac{5}{2}\left[\left(\frac{a}{2}-z\right)^2+a^2\right]^{3/2}2\left(\frac{a}{2}-z\right)}{\left[\left(\frac{a}{2}-z\right)^2+a^2\right]^{5}}\right)$$

$$z=0 \Rightarrow \frac{d^2\vec{B}}{dz^2}(0) = -\frac{3\mu_0 N i a^2}{2}\left(\frac{\left(\frac{5}{4}a^2\right)^{5/2} - \frac{5a^2}{4}\left(\frac{5}{4}a^2\right)^{3/2}}{\left(\frac{5}{4}a^2\right)^{5}} + \frac{\left(\frac{5}{4}a^2\right)^{5/2} - \left(\frac{5a^2}{4}\right)\left(\frac{5a^2}{4}\right)^{3/2}}{\left(\frac{5}{4}a^2\right)^{5}}\right) = \vec{0}$$

(9)



i, nL espiras

z'

$B(z) = ?$

$$dB = \frac{\mu_0 i' a^2}{2\left[a^2+(z-z')^2\right]^{3/2}} = \frac{\mu_0 i n a^2 dz'}{2\left[a^2+(z-z')^2\right]^{3/2}}$$

$$B = \frac{\mu_0 i n a^2}{2}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dz}{\left[a^2+(z'-z)^2\right]^{3/2}} = \frac{\mu_0 i n a^2}{2}\int_{-\frac{L}{2}-z}^{\frac{L}{2}-z}\frac{du}{\left[a^2+u^2\right]^{3/2}}$$

$u = z' - z$

$u = a\,tg\,\theta$

$du = a\sec^2\theta\, d\theta$

$$B = \frac{\mu_0 i n a^2}{2}\int_{-arctg\frac{L/2+z}{a}}^{arctg\frac{L/2-z}{a}}\frac{a\sec^2\theta\, d\theta}{a^3\sec^3\theta}$$

$\theta_1 = -arctg\,\frac{\frac{L}{2}+z}{a}$

$\theta_2 = arctg\,\frac{\frac{L}{2}-z}{a}$

$$B = \frac{\mu_0 n i}{2}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta = \frac{\mu_0 n i}{2}\left(\frac{L/2 - z}{\sqrt{(L/2-z)^2+a^2}} + \frac{L/2+z}{\sqrt{\left(\frac{L}{2}+z\right)^2+a^2}}\right)$$

$\theta_1 = -arcsen\,\frac{L/2+z}{\sqrt{(L/2+z)^2+a^2}}$

$\theta_2 = arcsen\,\frac{L/2-z}{\sqrt{\left(\frac{L}{2}-z\right)^2+a^2}}$

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$$z = -\frac{L}{2} \Rightarrow B\left(-\frac{L}{2}\right) = \frac{\mu_0 n i}{2}\left(\frac{L}{\sqrt{L^2+a^2}} + 0\right)$$

$$B\left(-\frac{L}{2}\right) = \frac{\mu_0 n i L}{2\sqrt{L^2+a^2}}$$

$$z = \frac{L}{2} \Rightarrow B\left(\frac{L}{2}\right) = \frac{\mu_0 n i}{2}\left(0 + \frac{L}{\sqrt{L^2+a^2}}\right)$$

$$B\left(\frac{L}{2}\right) = \frac{\mu_0 n i L}{2\sqrt{L^2+a^2}}$$

$$z = 0 \Rightarrow B(0) = \frac{\mu_0 n i}{2}\left(\frac{L/2}{\sqrt{L^2/4+a^2}} + \frac{L/2}{\sqrt{L^2/4+a^2}}\right)$$

$$B(0) = \frac{\mu_0 n i L}{\sqrt{L^2+4a^2}}$$

b) $$B(x) = \frac{\mu_0 n i}{2}\left(\frac{L/2 - x}{\sqrt{(\frac{L}{2}-x)^2+a^2}} + \frac{L/2 + x}{\sqrt{(\frac{L}{2}+x)^2+a^2}}\right)$$

$x \gg a$

$$B(x) = \frac{\mu_0 n i}{2}\left(\frac{\frac{L}{2}-x}{|\frac{L}{2}-x|} + \frac{\frac{L}{2}-x}{\frac{L}{2}+x}\right) = \frac{\mu_0 n i}{2}\left(\frac{L}{2}-x\right)\left(\frac{|\frac{L}{2}-x| + \frac{L}{2}-x}{|\frac{L^2}{4}-x^2|}\right)$$

$x \gg L$

$$B(x) = \frac{\mu_0 n i}{2}\left(\frac{-x}{\sqrt{x^2+a^2}} + \frac{x}{\sqrt{x^2+a^2}}\right) = 0$$

c) $L = 10a \Rightarrow \cancel{L \gg a}$

$$B(0) = \frac{\mu_0 n i\, 10\cancel{a}}{\sqrt{104\cancel{a^2}}} = \frac{10\mu_0 n i}{\sqrt{104}} = \frac{5\mu_0 n i}{\sqrt{26}}$$

$$B(x) = \frac{\mu_0 n i}{2}\left(\frac{5a-x}{\sqrt{(5a-x)^2+a^2}} + \frac{5a+x}{\sqrt{(5a+x)^2+a^2}}\right)$$

$$\frac{B(x)}{B(0)} = \frac{\sqrt{26}}{10}\left(\frac{5a-x}{\sqrt{(5a-x)^2+a^2}} + \frac{5a+x}{\sqrt{(5a+x)^2+a^2}}\right)$$

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(10)

$\hat{z}$

$O \quad R \quad \omega$

a) $di = \frac{dq}{t} = \frac{\omega\, dq}{2\pi} = \frac{\omega \sigma \cancel{2\pi} r\, dr}{\cancel{2\pi}} = \omega \sigma r\, dr$

$d\vec{B}_r = \frac{\mu_0 di}{4\pi} \cdot \frac{dl \,\mathrm{sen}\frac{\pi}{2}\hat{z}}{r^2} = \frac{\mu_0 i}{4\pi r}\, 2\pi r\, \hat{z}$

$dB = B_{r_\pi} = \frac{\mu_0 di}{2r}\hat{z} = \frac{\mu_0 \omega \sigma \cancel{r} dr}{2\cancel{r}}\hat{z}$

$\vec{B} = \frac{\mu_0 \omega \sigma R}{2}\hat{z}$

b) $d\vec{m} = di\, \vec{S} = \omega \sigma r\, dr\, \pi r^2 \hat{z} = \pi \omega \sigma r^3 dr\, \hat{z}$

$\vec{m} = \pi \int_0^R \omega \sigma r^3 dr\, \hat{z} \Rightarrow \vec{m} = \frac{\pi}{4}\omega \sigma R^4 \hat{z}$

(11)

a) $\oint \vec{B} \cdot d\vec{l} = \mu_0 i$

$B \oint dl = \mu_0 i$

$B = \frac{\mu_0 i}{2\pi r}$

$l = r\,\mathrm{tg}\,\theta$

$dl = r \sec^2\theta\, d\theta$

$dB_z = \frac{\mu_0 i}{4\pi} \cdot \frac{dl \cos\theta}{r^2} = \frac{\mu_0 i}{4\pi} \cdot \frac{\cancel{r \sec^2\theta}\, d\theta \cos\theta}{\frac{r^{\cancel{2}}}{\cancel{\cos^2\theta}}} = \frac{\mu_0 i}{4\pi r}\cos\theta\, d\theta$

$B_z = \frac{\mu_0 i}{4\pi r}\left[\mathrm{sen}\,\theta\right]_{-\frac{\pi}{2}}^{0}$

$B_z = \frac{\mu_0 i}{4\pi r} = \frac{B}{2}$

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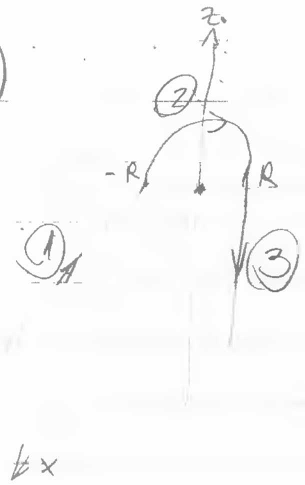
b)



(1): $\vec{B}_1 = \frac{\mu_0 i}{4\pi R}(-\hat{z})$

(3): $\vec{B}_3 = \frac{\mu_0 i}{4\pi R}(-\hat{x})$

(2): $\vec{B}_2 = \frac{\mu_0 i}{4\pi} \cdot \frac{1}{R^2} \int_0^{\pi R} dl \, (-\hat{x})$

$$B_2 = \frac{\mu_0 i}{4R}(-\hat{x})$$

$$\vec{B} = -\frac{\mu_0 i}{4R}\left[\left(\frac{1}{\pi}+1\right)\hat{x} + \frac{1}{\pi}\hat{z}\right]$$